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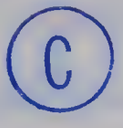
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THE UNIVERSITY OF ALBERTA

THE INFLUENCE OF COLUMN SHAPE ON THE BEHAVIOR OF FLAT PLATE SLABS

by



ROBERT JAMES KAVANAGH

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled THE INFLUENCE OF COLUMN SHAPE ON THE BEHAVIOR OF FLAT PLATE SLABS submitted by Robert James Kavanagh in partial fulfilment of the requirements for the degree of Master of Science.

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ABSTRACT

This investigation was confined to an elastic analysis of a typical interior panel of a continuous flat plate slab supported at intervals by non-deflecting columns. The variables considered were the column shape, the slab aspect ratio, Poisson's ratio and different types of loads. Using the method of finite differences the deflections, bending moments, twisting moments and shears were found at a large number of points on the panel.

An increase in the rectangularity of the column decreased the maximum deflection and bending moments. There was a smooth transition between two-way and one-way slab action as the ratio of the larger column dimension to the smaller column dimension increased to its limiting value.

The distribution of bending moments in the panel is compared with the design recommendations of ACI (318-63).

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CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

The design methods for reinforced concrete flat plate slabs have evolved from practice and are therefore essentially empirical. These procedures envision one type of slab only; that is a slab which carries the superimposed load by two-way action to square or circular columns with c/L ratios in the order of 1/18 to 1/20.

Economic and esthetic demands in multistory construction have caused variations in some flat plate slabs. These variations limit the applicability of the empirical design procedures. One such variation is an elongation of the support or column cross section.

1.2 Object

The object of this study was to determine the behavior of flat plate slabs supported by columns of varying aspect ratios. The aspect ratio of the column is defined as the ratio of its greatest lateral dimension to its least lateral dimension. It varies from a minimum when the slab is supported by a square column to a maximum when the slab is supported along two sides by a wall.

With different column aspect ratios the effects of a number of other variables upon the behavior of the slab were studied. These

variables included loading conditions, the ratio of slab spans in perpendicular directions, and Poisson's ratio.

1.3 Scope

The slab investigated was a typical interior panel of an infinite flat plate supported at intervals by nondeflecting columns. Because of symmetry only one quarter of the typical panel had to be considered for analysis.

Only loads perpendicular to the plate were considered. The elastic deflections, bending moments and twisting moments at a large number of points on the panel were determined for each of the variables examined. The shear distribution around the perimeter of the column was also examined.

1.4 Historical Review

Considerable research, both analytical and experimental, has been carried out on flat plate concrete slabs. Despite this, there exists in the literature a void with respect to the influence of the aspect ratio of the column upon the behavior of the slab.

Timoshenko (1) examined the problem briefly. However his analysis was restricted to the case where the aspect ratios of the columns and the slab are equal. The application of this to most problems encountered in practice is limited.

The limitations of the present design procedures are clearly recognized and the need for this type of investigation has been recommended in the Commentary on Building Code Requirements for

Reinforced Concrete (ACI 318-63), (2).

1.5 Notation

a	=	span length of one panel in x-direction, centre to centre distance between columns
b	=	span length of one panel in y-direction, centre to centre distance between columns
c/L	=	ratio of support size to the centre to centre distance between columns
E	=	modulus of elasticity
F	=	a factor used in ACI building code to increase the safety factor for flat plate slab construction
D	=	$\frac{Et^3}{12(1-\mu^2)}$, a measure of slab stiffness
h, h_x,h_y	=	distance between node points on finite difference grid with respect to the coordinate directions.
M_o	=	numerical sum of positive and negative moments at critical design sections
m_x	=	bending moment intensity at a point about y-axis
m_y	=	bending moment intensity at a point about x-axis

m_{xy}	=	twisting moment intensity at a point
q	=	uniformly distributed load per unit of area
t	=	slab thickness
x,y,z	=	rectangular reference coordinates
w	=	vertical deflection of slab
V	=	intensity of shear at a point
μ	=	Poisson's ratio

CHAPTER II

METHOD OF ANALYSIS

2.1 Introduction

The governing differential equation for the bending of medium thick, elastic plates is:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

The assumptions used in formulating this equation have been stated in the literature (1,3).

A solution of the above equation for the values of deflection, w, satisfying the particular boundary conditions of a given plate yields through its derivatives the moments and shears within the plate. For this investigation the equation was solved by the method of finite differences.

Calculations were carried out on the IBM 7040 computer at the University of Alberta.

2.2 Finite Difference Method of Analysis

To apply the finite difference method to physical problems of a continum such as a plate, the plate surface is laid off in a grid and a general fourth order difference equation is written for each node point in the grid. The difference equation for any specific

point expresses the deflection of that point in terms of the deflections of adjacent points. Solving the set of difference equations for the entire grid simultaneously yields the value of the deflection at each individual grid point. These deflections may then be substituted into other difference expressions to determine moments and shears.

In the boundary regions, points lying outside of the plate being considered are generally involved. These points can either be eliminated by expressing them in terms of interior points by means of the equations specified by the prescribed boundary conditions or the additional equations can be written so that the total number of equations is equal to the number of unknown deflections.

The method of finite differences has been explained in detail by Ang and Prescott (3). Appendix A contains examples of the difference equations used for the deflections, moments and shears in this analysis.

2.3 Solution of Equations

One characteristic of a finite difference solution is that the difference equation for any particular point includes only points nearby while the coefficients of the remaining terms in the equation are zero. The result is that all non-zero coefficients fall within a distinct band around the main diagonal of the array as shown in Figure 2.1.

For this analysis of a total of 441 simultaneous equations had

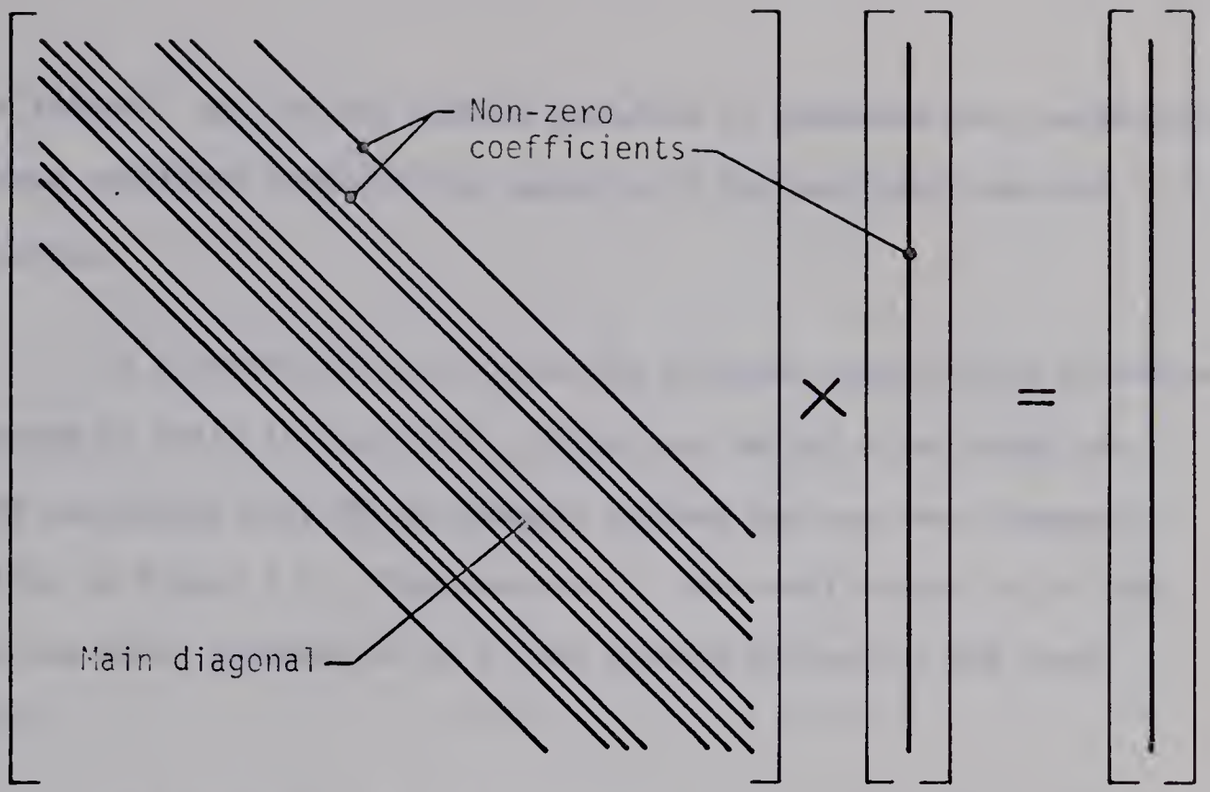


FIGURE 2.1 FINITE DIFFERENCE MATRIX IN CONVENTIONAL FORM

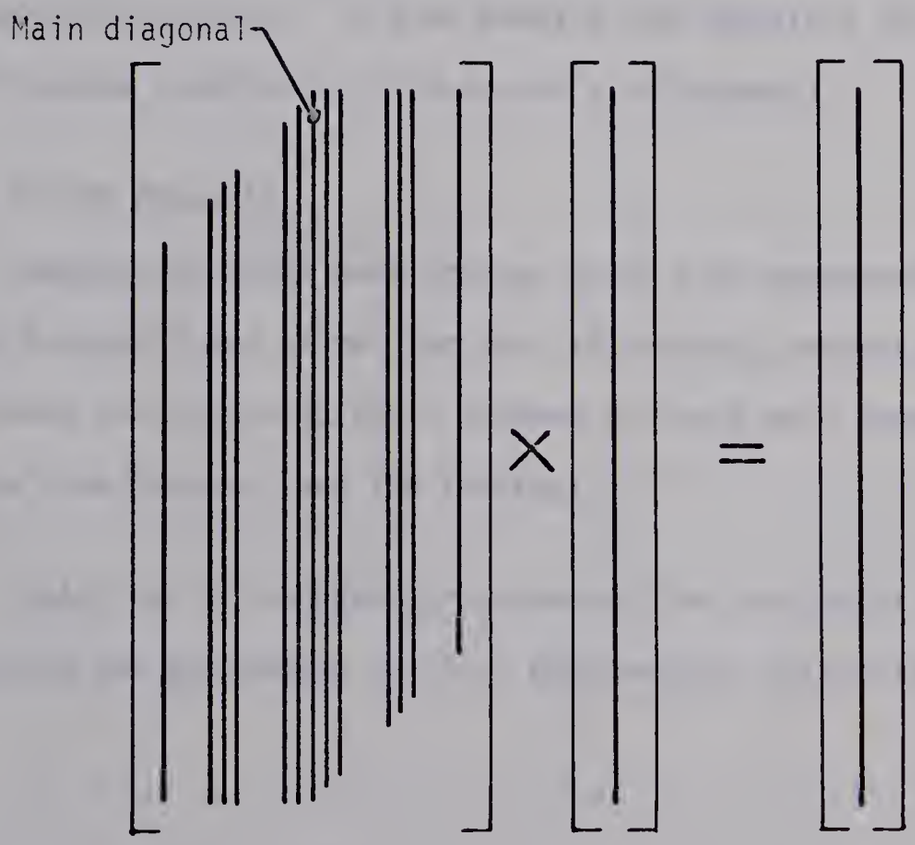


FIGURE 2.2 FINITE DIFFERENCE MATRIX IN DIAGONALLY SUBSCRIPTED FORM





to be solved. The 195,000 numbers required to represent the coefficients in these equations exceeded the capacity of the available computer facilities.

To circumvent this problem the diagonal subscripting technique developed by White (4) was used. Using this method a new array was formed containing only those elements between the non-zero diagonals as shown in Figure 2.2. This resulted in the coefficients of the 441 equations being represented by 37,600 numbers instead of the usual 195,000.

The equations, in the diagonally subscripted form, were solved by a variation of the Gauss - Elimination Method develped by Vollan (5). Basically, the variation takes into account the diagonal subscripting of the array. It also permits the equations for several different loading conditions to be solved simultaneously.

2.4 Form Of The Analysis

Computer programs were written which both generated the difference equations and solved for the deflections, moments and shears. Any particular solution required a minimum of input data consisting only of the slab geometry and the loadings.

Copies of the computer programs and the results of the analysis are filed with the Department of Civil Engineering, University of Alberta.

2.5 Accuracy of Solution

The major problem in such an analysis is obtaining the correct solution to the large number of simultaneous equations. Computers only retain a predetermined number of digits and there may be a loss in significance when nearly identical numbers are subtracted, when a small number is multiplied by a small number, when a small number is divided by a large number and when truncation occurs.

The analysis of a square column supporting a square slab afforded an extra degree of symmetry along the diagonal by which the accuracy of the solution could be checked. While some symmetrical deflections, moments and shears agreed to three significant figures, they generally agreed to only two significant figures.

The sum of the bending moments across any two parallel section lines through the panel a distance l apart must satisfy statics, i.e. $\frac{wl^2}{8}$. For each of three different column aspect ratios this check on the accuracy of the results was performed on five different strips of the panel. The sum of the moments were from 1.2 to 6.7% low in each case. The largest error occurred in the case of the largest column aspect ratio.

As a second check the sum of the shears around the column was compared to the superimposed load for five different column aspect ratios. The sum of the shears was 1 to 3% low, the largest error again occurring in the case of the largest column aspect ratio.

Previous work by Vollan indicated that the grid division chosen for this analysis should yield errors less than 1%. It was

felt that the additional error stemmed from the fact that each number could only be represented by eight digits in the over 1.5 million calculations required for the solution of each set of equations. Had the capacity of the computer facilities been twice as large so that each number could be represented by sixteen instead of eight digits the error would probably have been reduced considerably.

CHAPTER III

OUTLINE OF INVESTIGATION

3.1 Introduction

This investigation was restricted to the analysis of a typical interior bay of a continuous flat plate slab. Due to symmetry along the column lines and the centre lines a solution was obtained by considering only one quarter of the panel as shown in FIGURE 3.1. Consistent with a series of limitations discussed later the effects of a number of variables upon the slab behavior were examined. Over one hundred combinations of variables were considered and the total was limited only by the time available.

3.2 Selection Of Grid

A minimum c/L ratio of 0.05 was chosen for this investigation. This corresponded to a 1'-0 wide column in a 20'-0 span which approximates the proportions of existing reinforced concrete flat plate construction. To facilitate analysis the column perimeter was always defined by grid points and a minimum column dimension of two grid divisions was chosen. With these criteria the quarter panel was divided into a 20 x 20 grid containing a total of 441 node points.

3.3 Boundary Conditions

Points falling outside the quarter panel were involved in the difference equations for the boundary regions. Along the column and

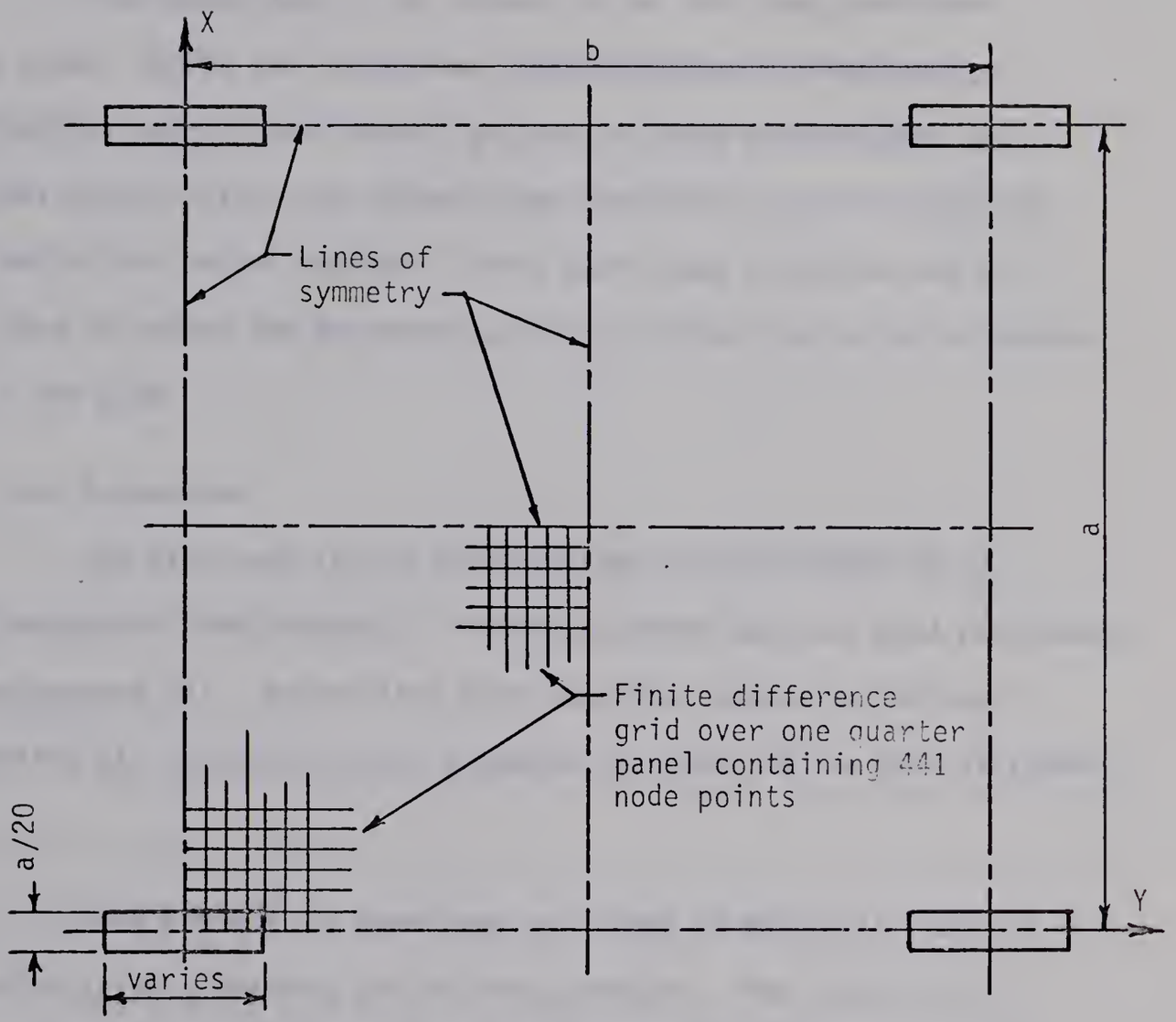


FIGURE 3.1 ARRANGEMENT OF FLAT PLATE SLAB INVESTIGATED



panel centrelines these were related to interior points by the zero slope and zero shear conditions existing along these lines of symmetry.

The connection of the column to the slab was considered fully fixed. While the column was also considered non-deflecting hypothetical points with virtual deflections were superimposed upon the real points within the column cross section so that the slope at the face of the column was zero. This zero slope condition was in turn used to relate the hypothetical points within the column to points within the slab.

3.4 Slab Properties

The slab used in the investigation was considered to be both homogeneous and isotropic. This assumption has been used previously by Westergaard (6). Rather than give numerical values to the slab properties all deflections were expressed in terms of the slab stiffness, D.

Both 0.0 and 0.2 were used as values of Poisson's ratio in the calculation of bending and twisting moments. The value of 0.2 approximates Poisson's ratio for the high-strength concretes normally used in flat slab construction.

3.5 Aspect Ratio Of Column

The primary variable in this analysis was the aspect ratio of the column. Five aspect ratios were considered; 1, 4, 8, 12, 16 corresponding respectively to c/L ratios of 0.05, 0.2, 0.4, 0.6 and

0.8 in the y-direction. In the x-direction the width of the column was kept constant with the c/L ratio maintained at 0.05.

3.6 Aspect Ratio Of Slab

Four slabs of differing aspect ratios were examined. The minimum ratio of 1.0 corresponded to the most common type of slab with a square panel. The maximum ratio used was 1.5. Solutions were also obtained for the intermediate ratios of. 1.2 and 1.33. The value of 1.33 corresponds to the largest aspect ratio for which the empirical flat slab design procedures of the ACI Code (7) can be used.

Data could not be obtained for aspect ratios larger than 1.5 since during the course of these solutions numbers smaller than those which the computer could handle were generated.

3.7 Loading Conditions

Four basic loading conditions were used. These were a uniformly distributed load, two line loads (one along each panel centre line), and a concentrated load at the centre point of the panel.

The concentrated load was applied over an area a/20 x b/20 in each case. This is similar to a concentrated load being applied over an area of one square foot on a 20'-0 x 20'-0 panel.

CHAPTER IV

PRESENTATION AND DISCUSSION OF RESULTS

4.1 Introduction

The numerical results obtained in this analysis included deflections, bending moments, twisting moments and the shears around the column. Each of the above, when applicable, was calculated for every node point on the panel. However, to illustrate general behavior only a small portion of this data is presented.

The symmetry of the typical interior panel enabled the presentation of the results by considering only one half of any section through the panel. FIGURE 4.1 illustrates the system used to identify section lines.

Bending moments are presented for $\mu = 0.0$ and $\mu = 0.2$. However, the results for $\mu = 0.0$ are only intended to help clarify the general behavior. Most results are presented for $\mu = 0.2$ to simulate the behavior of an actual reinforced concrete flat plate. Unless otherwise indicated all results reported graphically are for $\mu = 0.2$.

The results are presented in terms of a right-handed coordinate system. Positive deflections are downward, positive bending moments cause compression on the top of the slab, positive shear causes

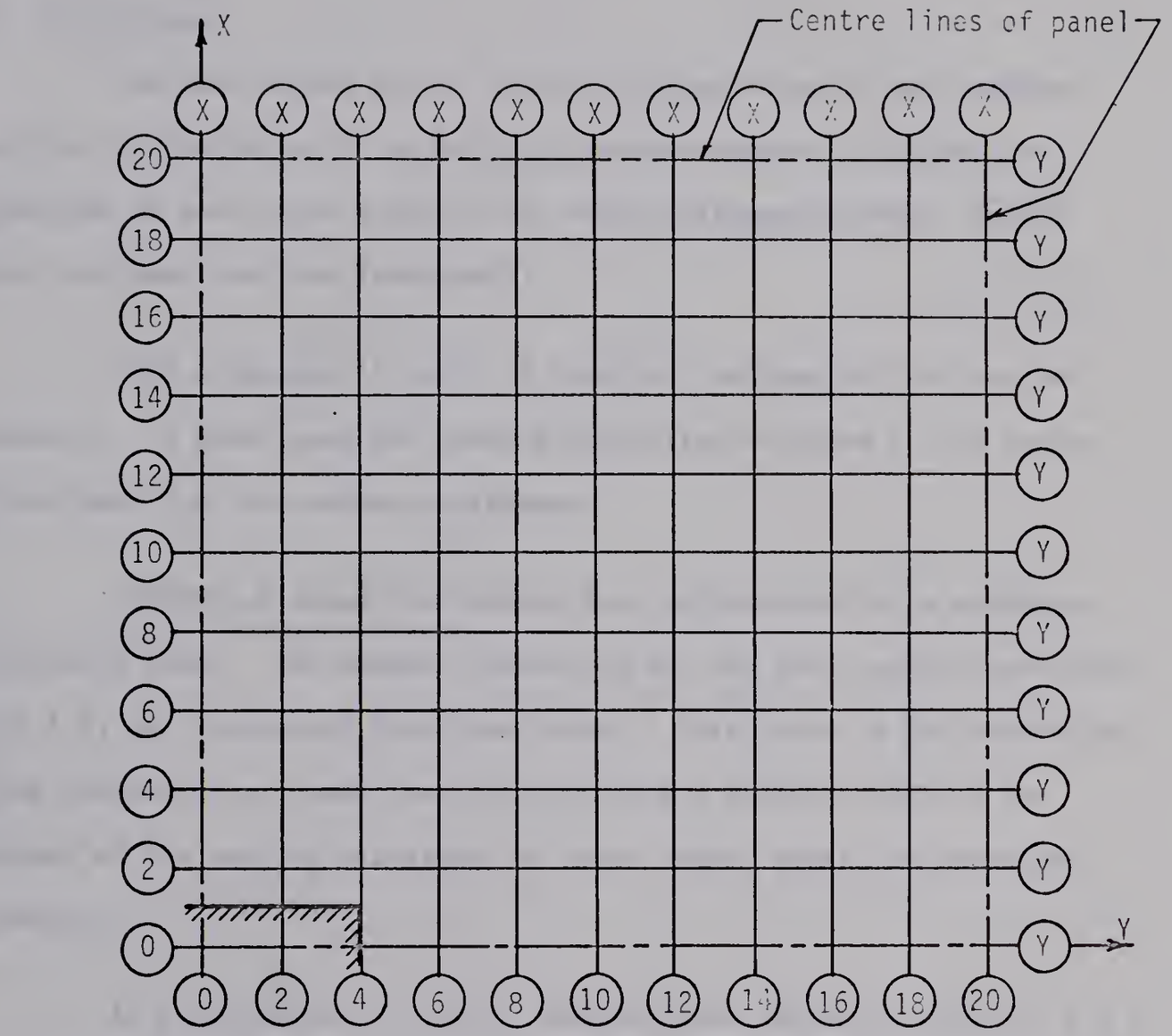
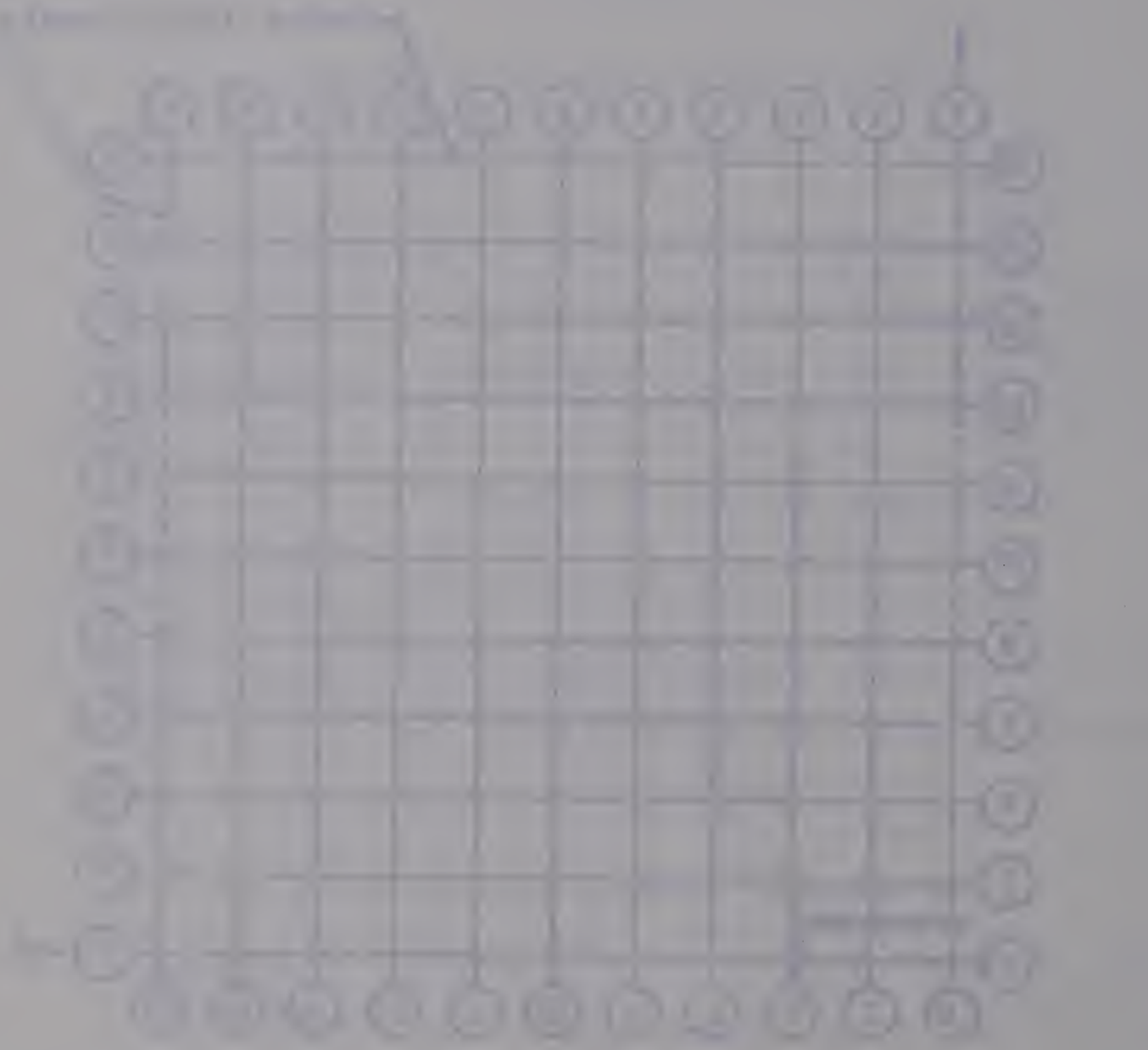


FIGURE 4.1 IDENTIFICATION OF SECTION LINES THROUGH ONE QUARTER PANEL



a compressive load in the column and positive twisting moments increase the compression force in the column.

4.2 Deflections

For each column aspect ratio, slab aspect ratio and loading condition the solution of the 441 simultaneous equations yielded the deflection at every node point on the finite difference grid. In all cases they were positive (downward).

From a designer's point of view only maximum deflections are important. In every case the maximum deflection occurred at the centre of the panel for the loading considered.

FIGURE 4.2 shows the maximum slab deflections for a uniformly distributed load. The maximum deflection for the wall support condition, c/L = 1.0, was calculated from beam theory. This plots on the projection of the curves for c/L less than 1.0 providing a further check on the accuracy of the results calculated by plate theory using the numerical procedure.

At a c/L ratio of 0.05 the maximum slab deflection for b/a = 1.5 is three times that of b/a = 1.0. As the column aspect ratio increases the difference in maximum slab deflections decreases. For all b/a ratios the maximum slab deflections approach asymptotically to the fixed beam deflection, $2.12 \times 10^{-3}\ qa^4/D$ when the slab is supported on two sides by a wall.

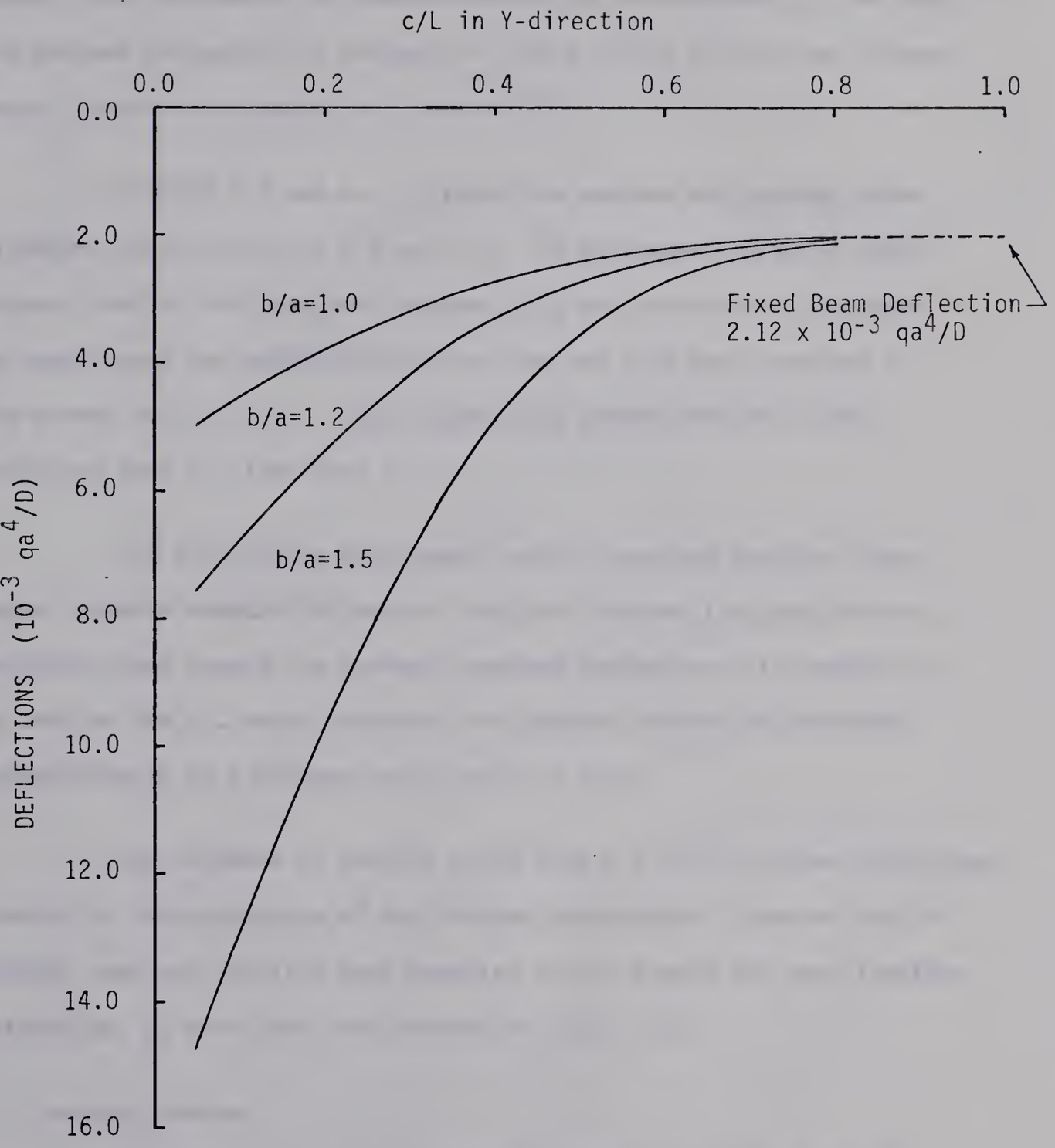


FIGURE 4.2 MAXIMUM SLAB DEFLECTIONS
UNIFORM LOAD; b/a=1.0,1.2,1.5

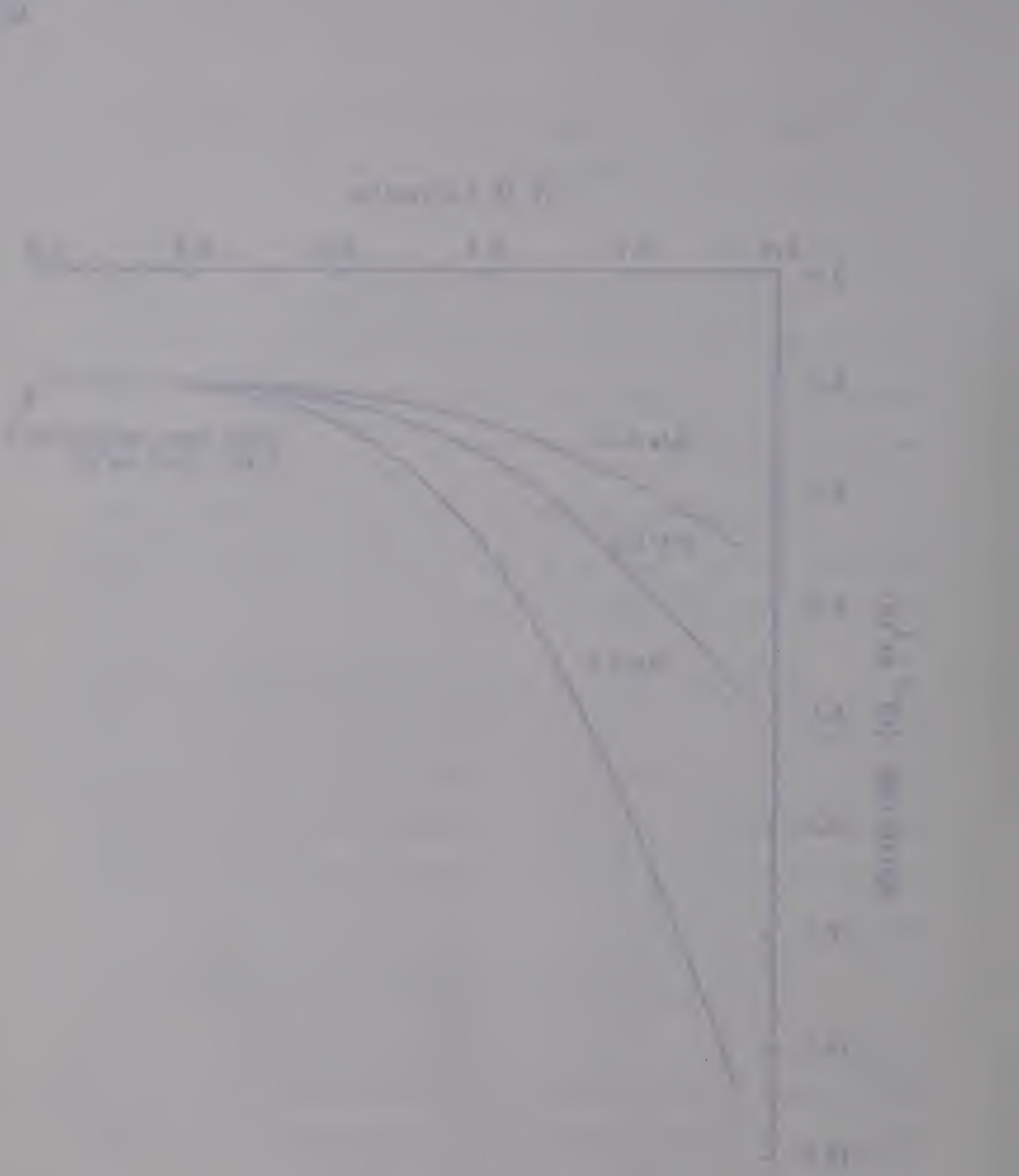


FIGURE 4.2 also shows how even a small increase in the column aspect ratio decreases the maximum deflection significantly. For b/a = 1.5 the maximum deflection is reduced to $9.62 \times 10^{-3}\ q\ a^4/D$ as the column aspect ratio is increased to 0.2 from 0.05.

FIGURES 4.3 and 4.4 indicate the maximum deflections under different loads for b/a = 1.0 and 1.5. In all cases the total superimposed load on the slab was the same. It was possible to calculate by beam theory the maximum deflection for the line load parallel to the y-axis for c/L = 1.0. Once again this agreed with the curves projected from c/L less than 1.0.

For both of the slab aspect ratios examined the line loads caused greater maximum deflections than the uniform load and the concentrated load caused the greatest maximum deflection. For each type of load as the c/L ratio increased the maximum deflection decreased asymptotically to a minimum value at c/L = 1.0.

The increase in the b/a ratio from 1.0 to 1.5 caused significant changes in the magnitudes of the maximum deflections. However for the uniform load and the line load parallel to the y-axis the same limiting deflection, in each case, was reached at c/L = 1.0.

4.3 Bending Moments

FIGURES 4.5 to 4.19 illustrate the general slab behavior with respect to bending moments. The plots of moment intensities at individual points versus c/L in the y-direction provide a further check

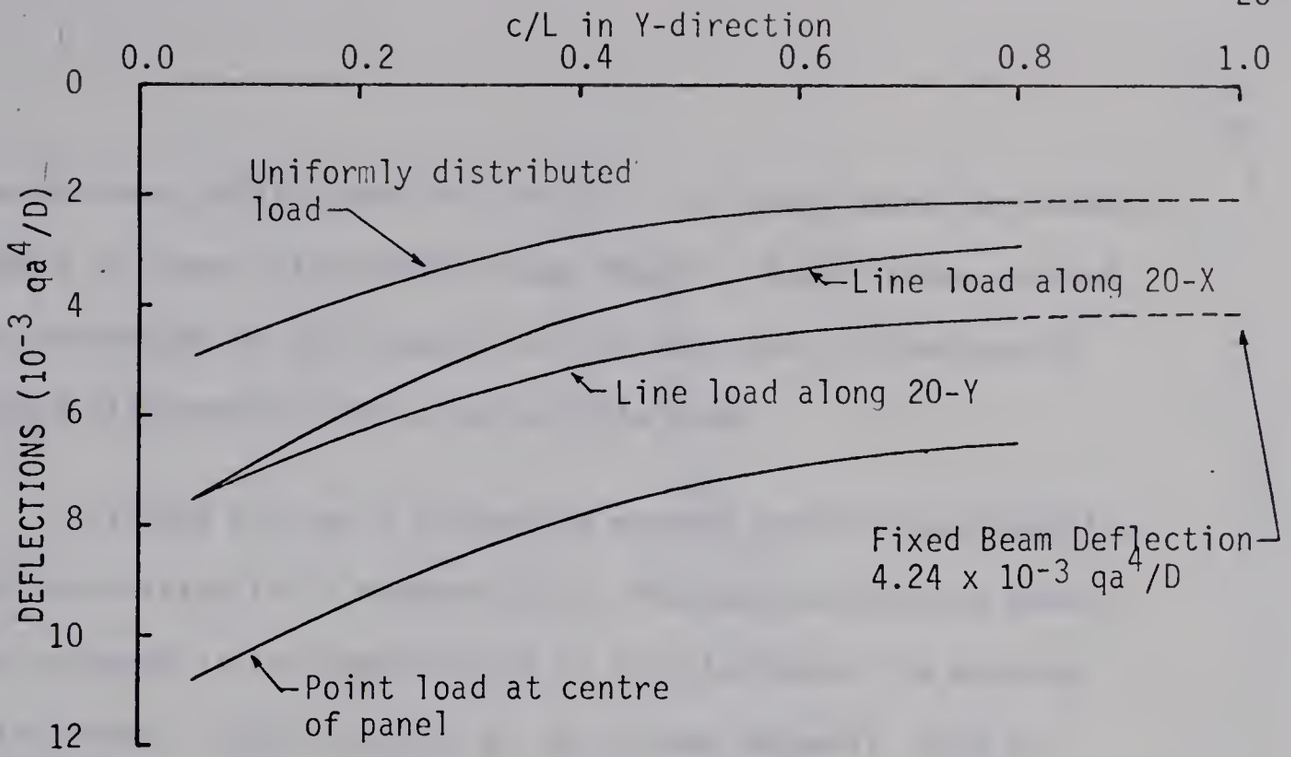


FIGURE 4.3 MAXIMUM SLAB DEFLECTIONS FOR DIFFERENT TYPES OF LOAD, b/a=1.0

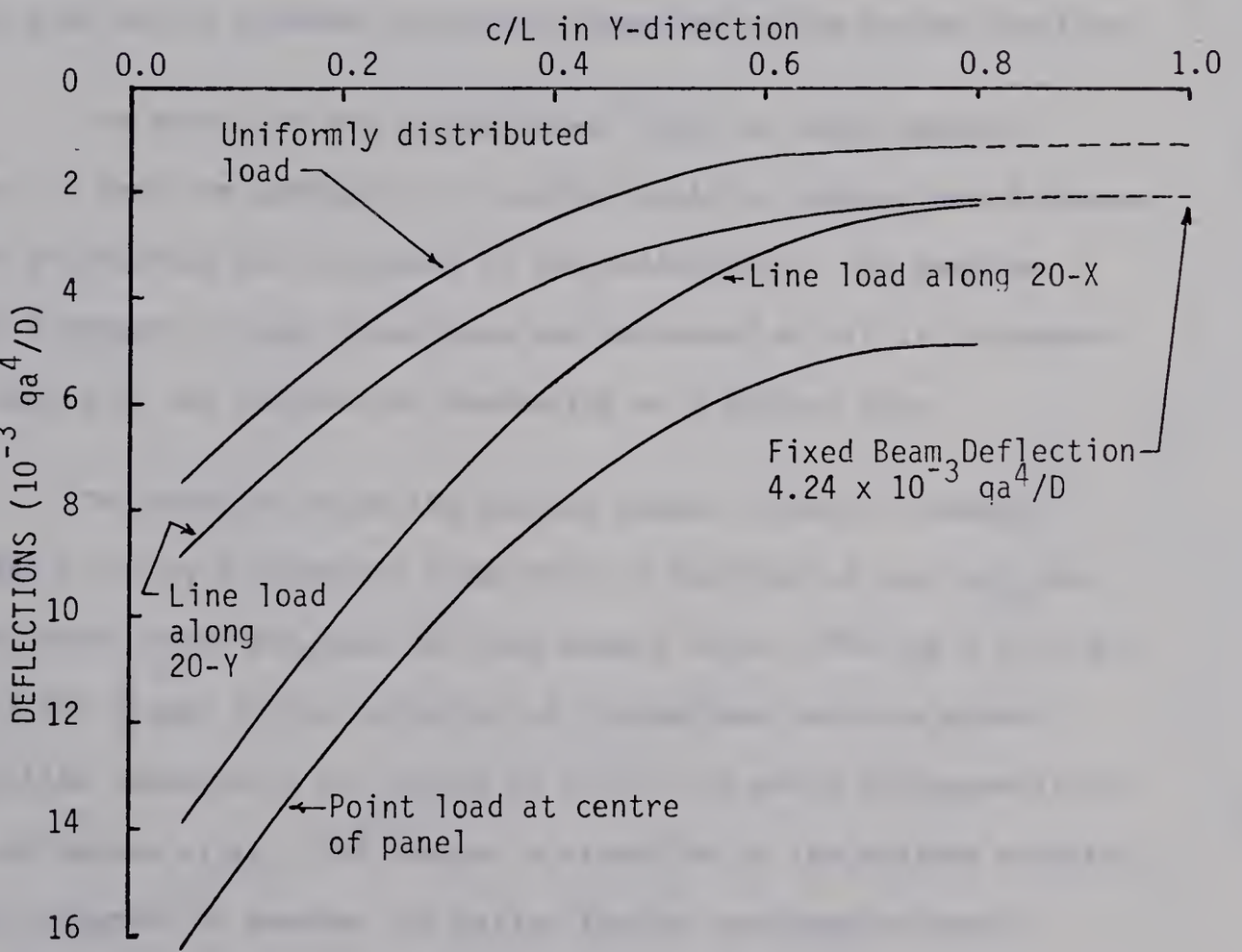
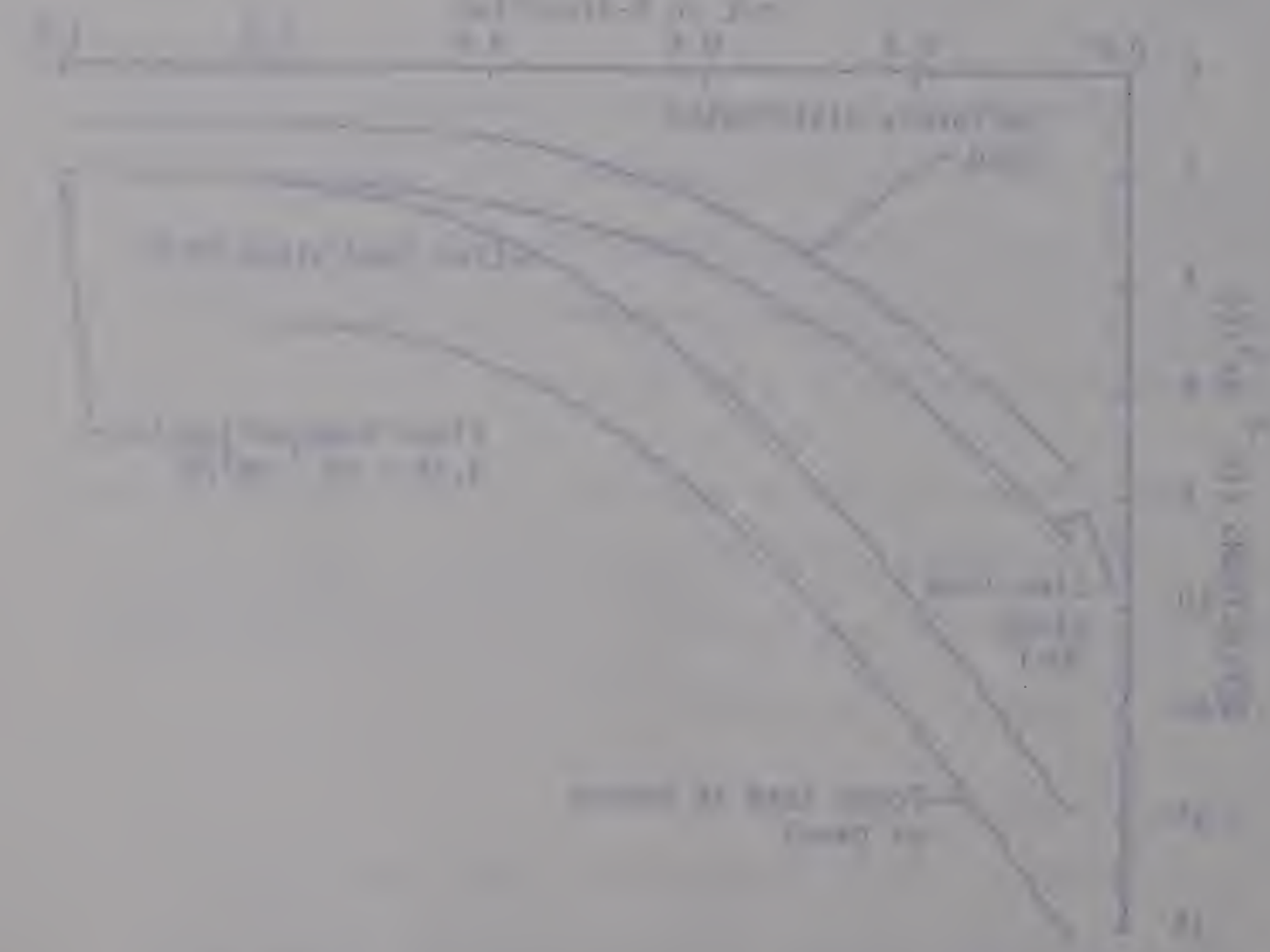


FIGURE 4.4 MAXIMUM SLAB DEFLECTIONS FOR DIFFERENT TYPES OF LOAD, b/a=1.5



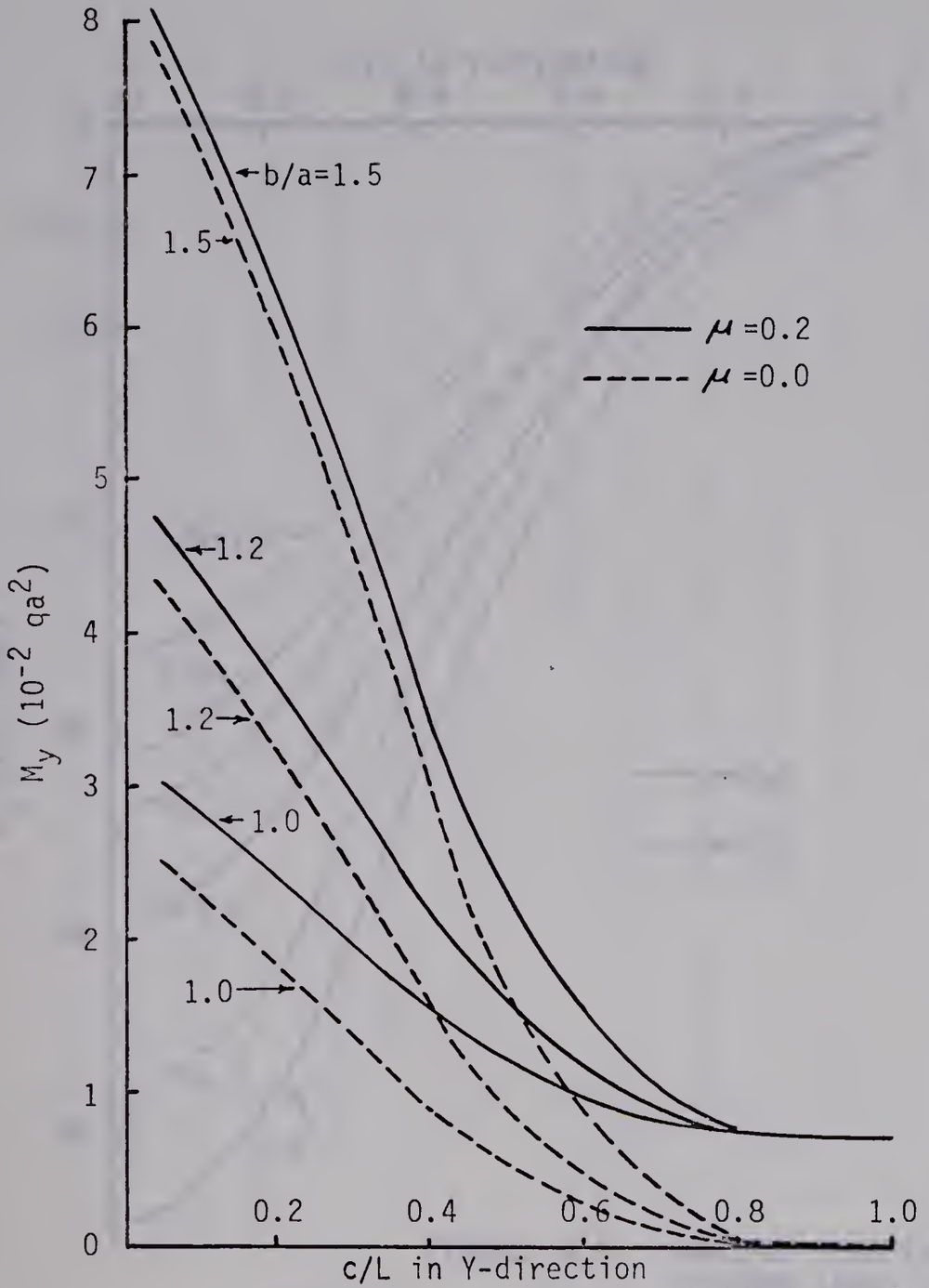


on the accuracy of the numerical results. In these cases the moments for c/L = 1.0 were calculated by beam theory. These values plotted on the projection of the results for c/L less than 1.0 calculated by finite differences with no perceptible error.

FIGURES 4.5 and 4.6 show the maximum positive and negative moment intensities for a uniform load. The maximum positive moment always occurred at the centre point of the slab while the maximum negative moment always occurred at the column corners. Thus on FIGURE 4.6 any one curve does not represent the moment at a single point on the slab but at a number of points depending on the column position.

The effect of the column aspect ratio is quite apparent. As the c/L ratio is increased the maximum positive moments are decreased in the y-direction and increased in the x-direction. The maximum negative moments in both directions are decreased as c/L is increased: the moments in the y-direction decreasing at a greater rate.

The point at which the maximum moment intensity changes from the y to the x-direction seems to be a function of not only the column aspect ratio but also the slab aspect ratio. For b/a = 1.0, 1.2, and 1.5 the change in the direction of the maximum positive moment intensities occurred at c/L ratios of 0.05, 0.28 and 0.42 respectively for both values of μ. The change in direction of the maximum negative moments occurred at greater c/L ratios for the rectangular panels. A simple relationship for this condition does not appear to exist.



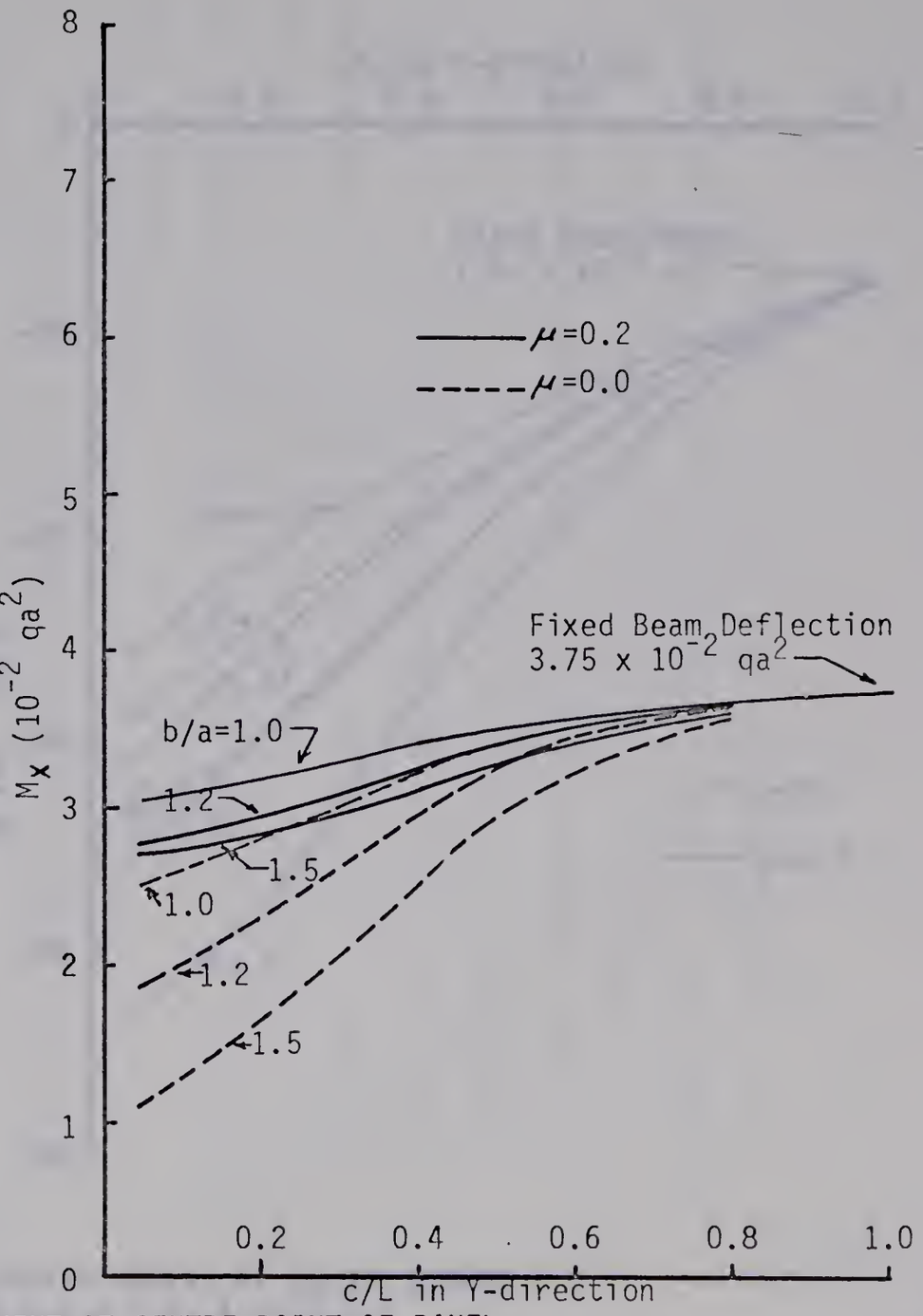
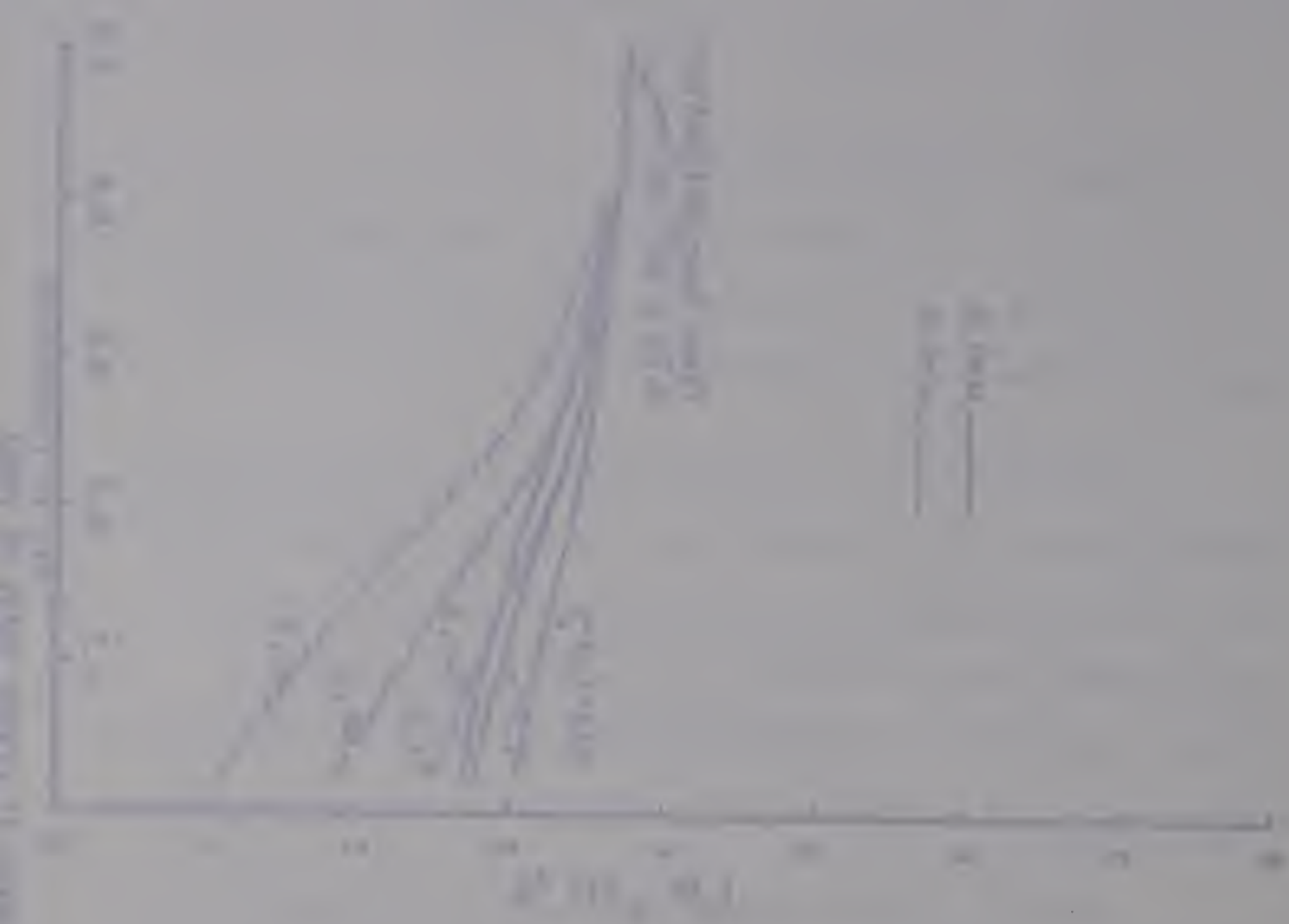
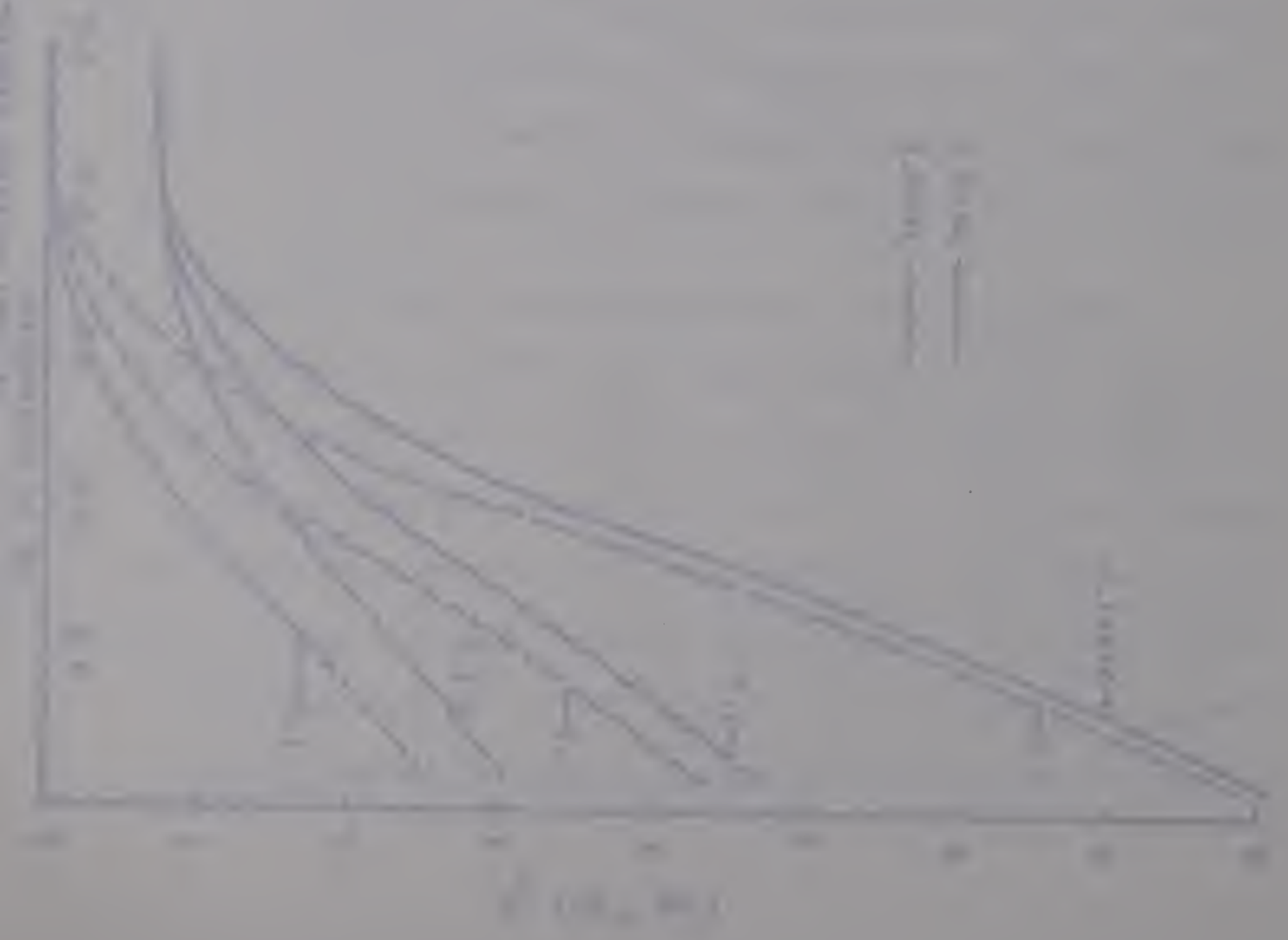
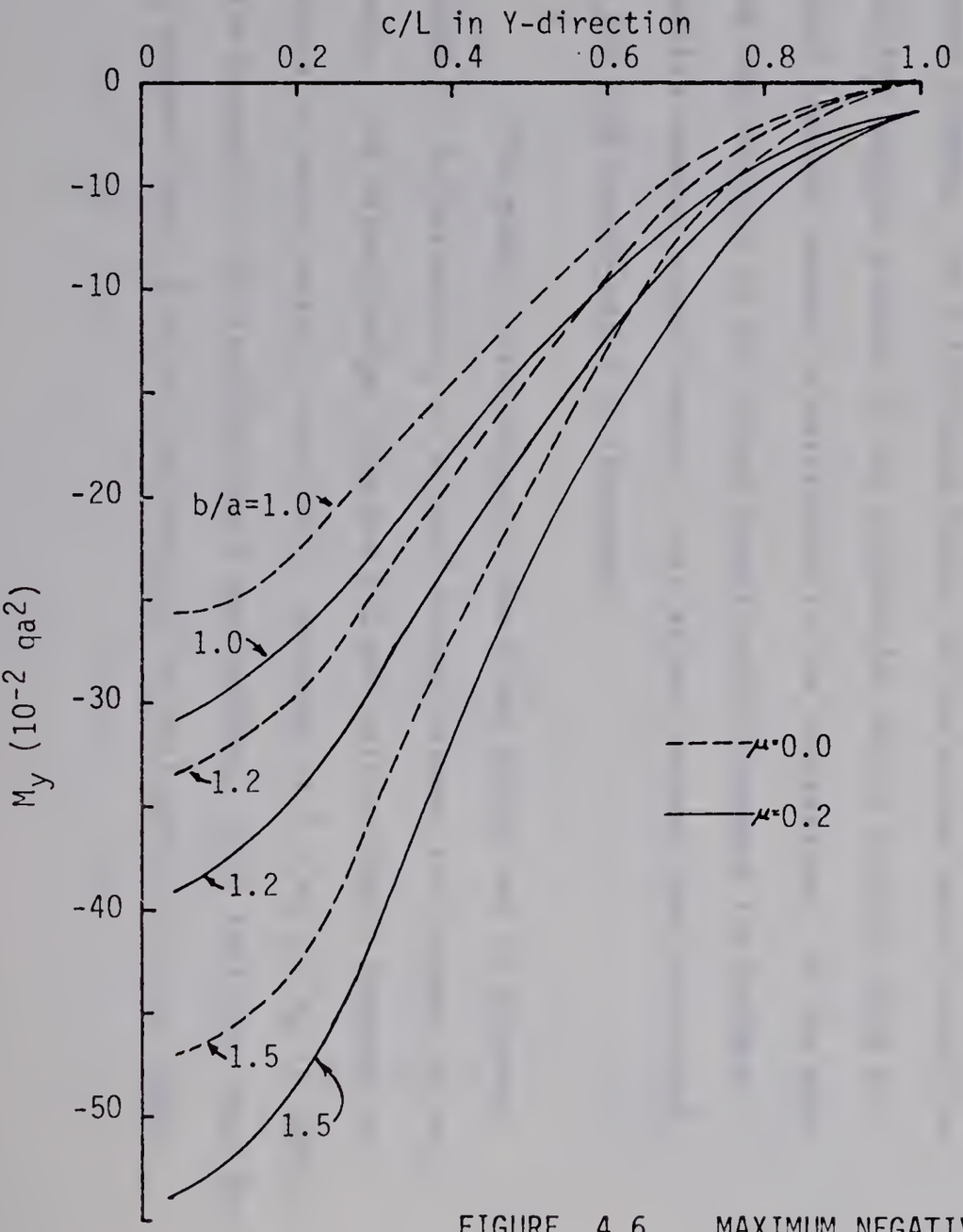


FIGURE 4.5 MAXIMUM POSITIVE BENDING MOMENT AT CENTRE POINT OF PANEL
UNIFORM LOAD; b/a=1.0,1.2,1.5;μ=0.0,0.2







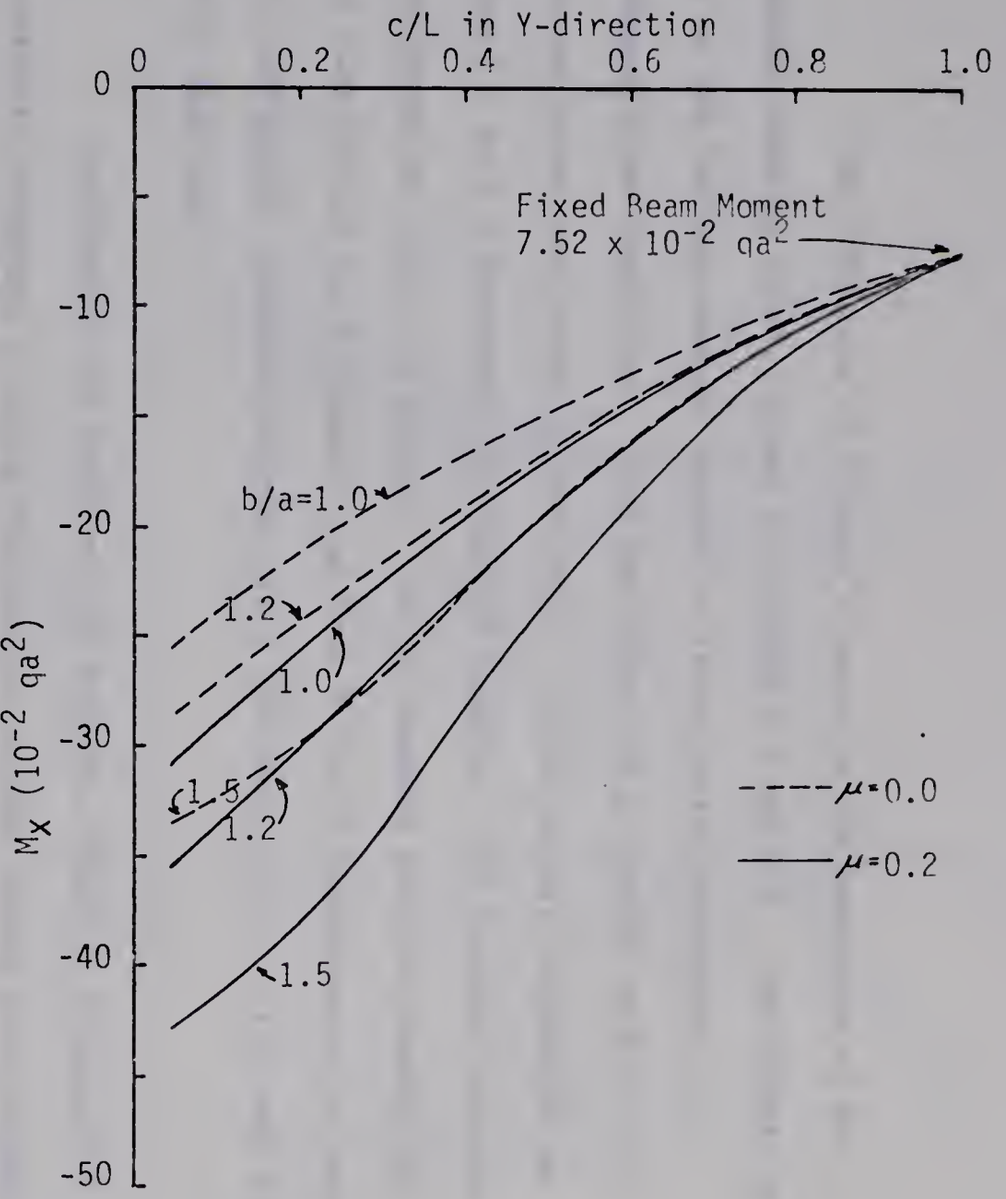
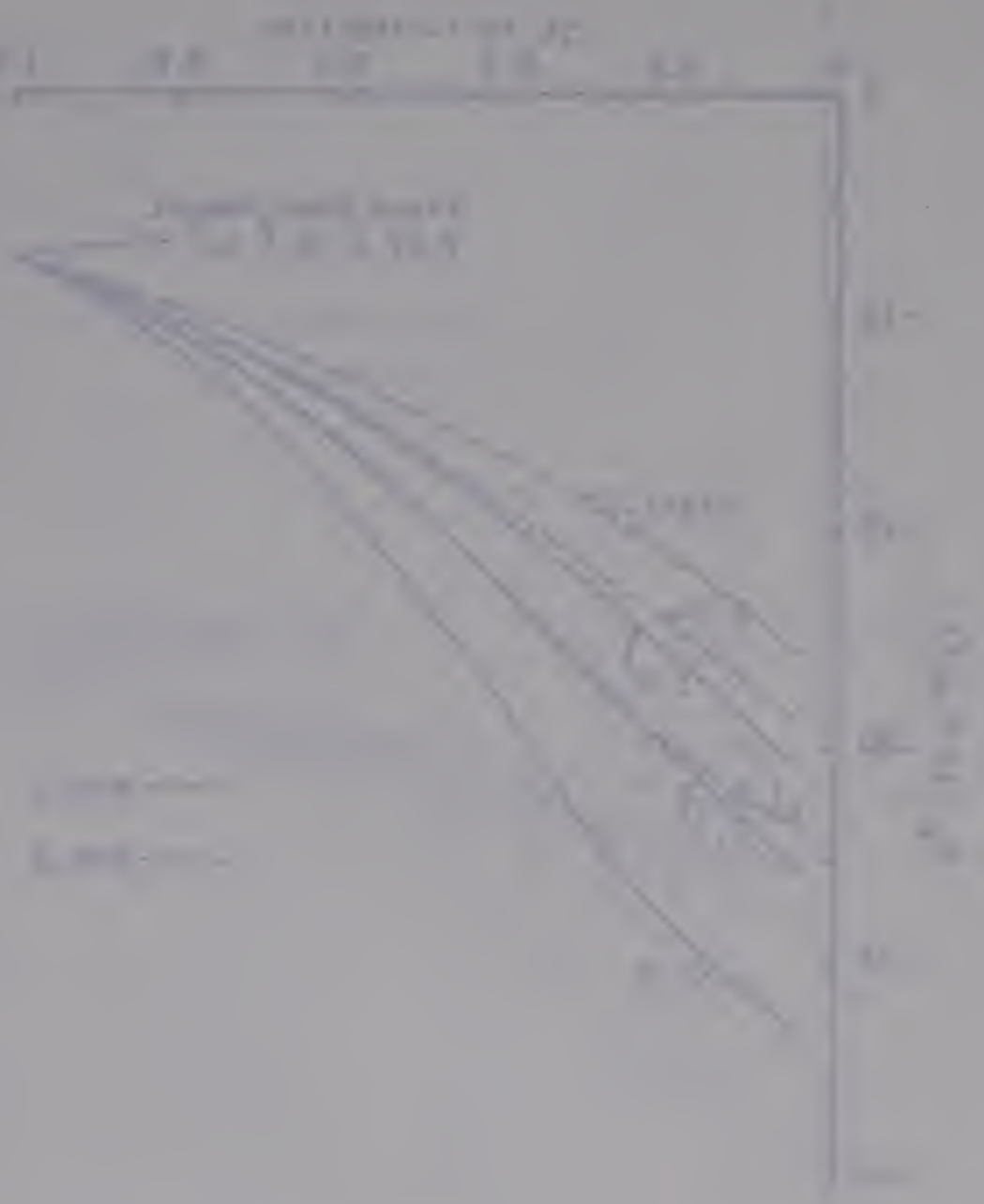


FIGURE 4.6 MAXIMUM NEGATIVE BENDING MOMENT AT COLUMN CORNER
UNIFORM LOAD; b/a=1.0,1.2,1.5; μ=0.0,0.2

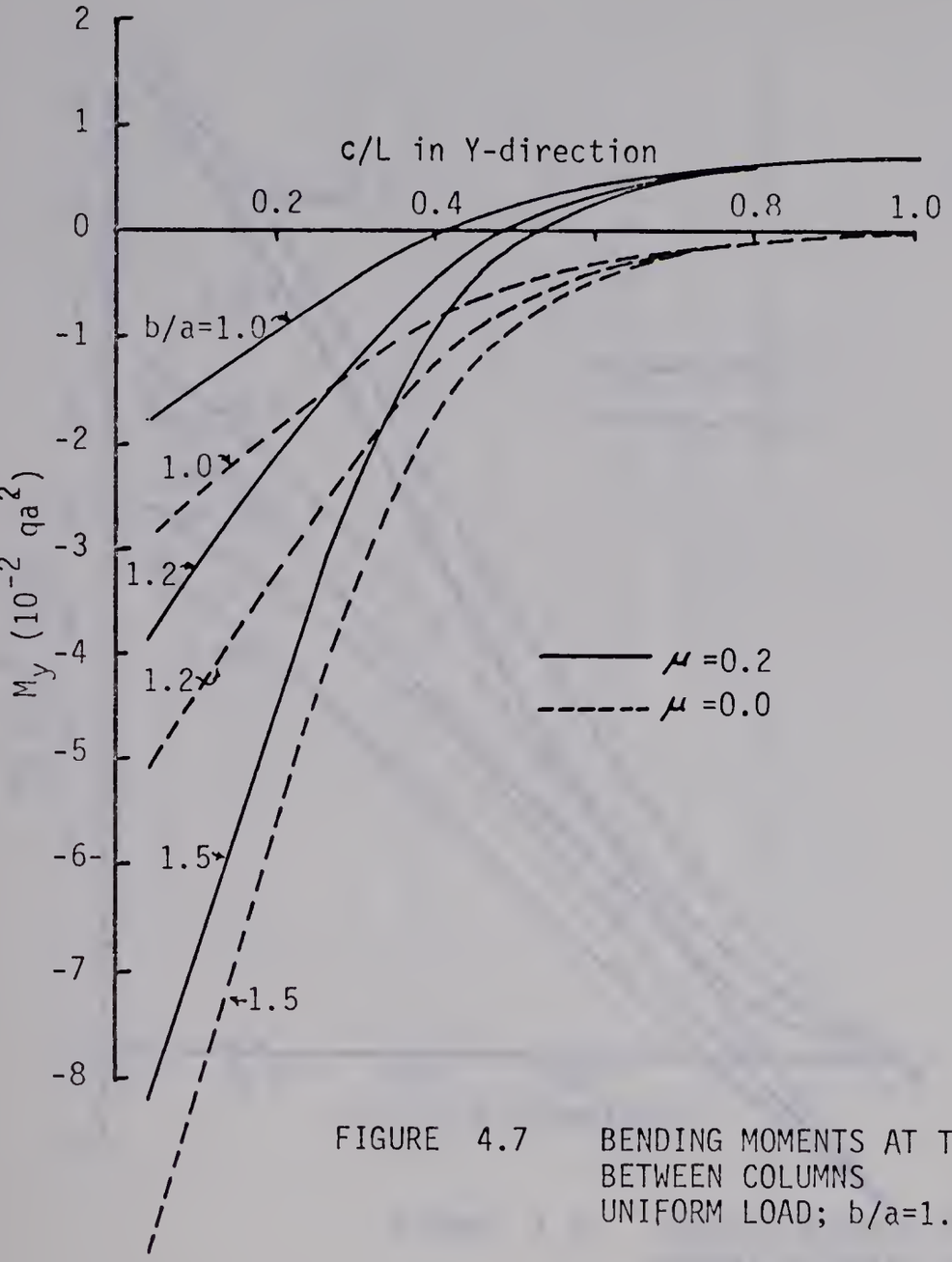




The influence of Poisson's ratio on the magnitude of the maximum moments is also shown in FIGURES 4.5 and 4.6. The effects of μ on the bending moments depends on the relative curvatures in perpendicular directions. Thus the effect of μ depends on both the b/a ratio and the column aspect ratio. In all cases, increasing μ from 0.0 to 0.2 increased the maximum moments. The increase was more pronounced at lower c/L ratios. For the maximum positive moments in the x-direction the increase in μ from 0.0 to 0.2 substantially decreased the difference in moment between c/L = 0.05 and c/L = 1.0 for each b/a ratio examined.

Considering the maximum slab moments, there is evidence of a definite advantage in using rectangular columns rather than conventional square columns. By a judicious choice of the column aspect ratio the positive bending moments in the y-direction can be limited while an almost constant moment is maintained in the x-direction. At the same time any increase in the column aspect ratio decreases the maximum negative moments significantly. This effect becomes more pronounced as the slab aspect ratio is increased.

The moment intensities at two other points are of interest. FIGURE 4.7 illustrates m_x at a point midway between the columns on the x-axis. The effects of μ, b/a and c/L are self-evident. However one feature is worth noting. For c/L in the range of 0.4 to 0.5 m_x is within a few percent of the limiting fixed beam moments. This indicates that for c/L greater than 0.4 to 0.5 the behavior of that portion of the slab



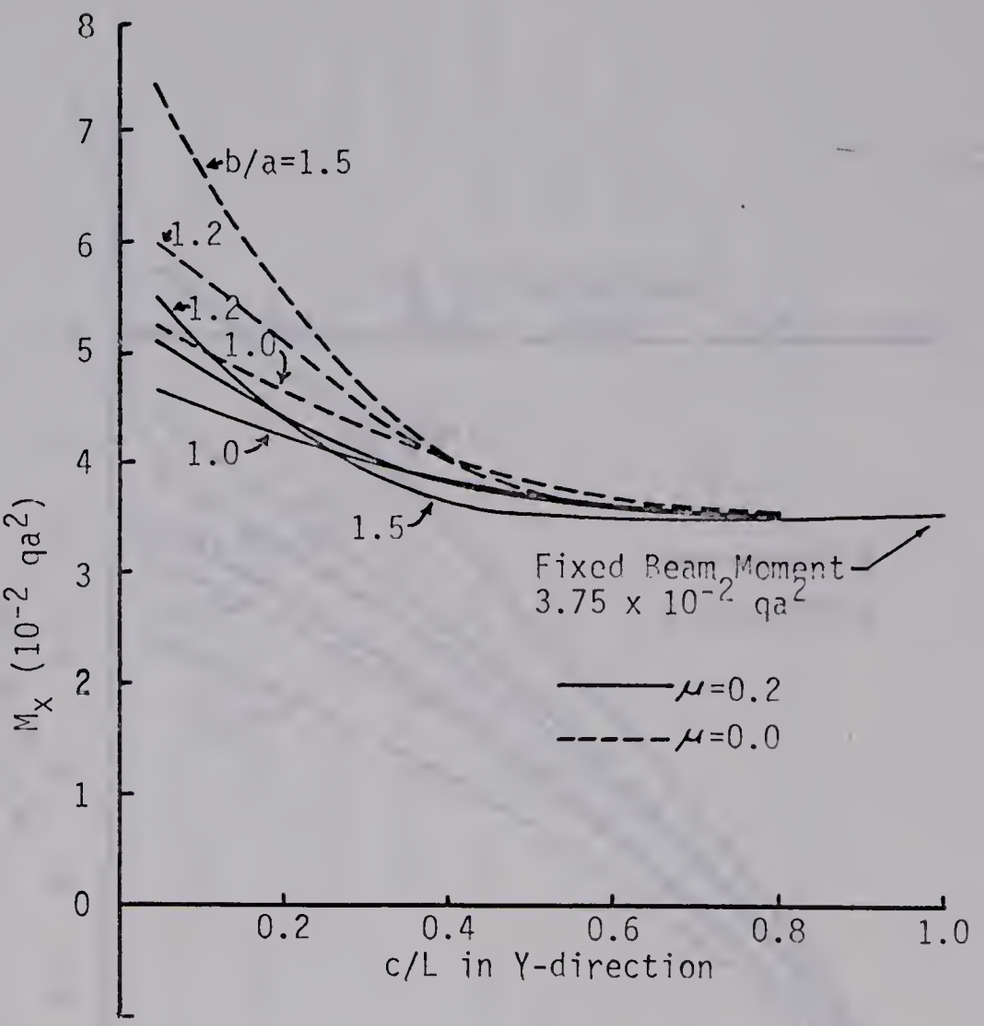
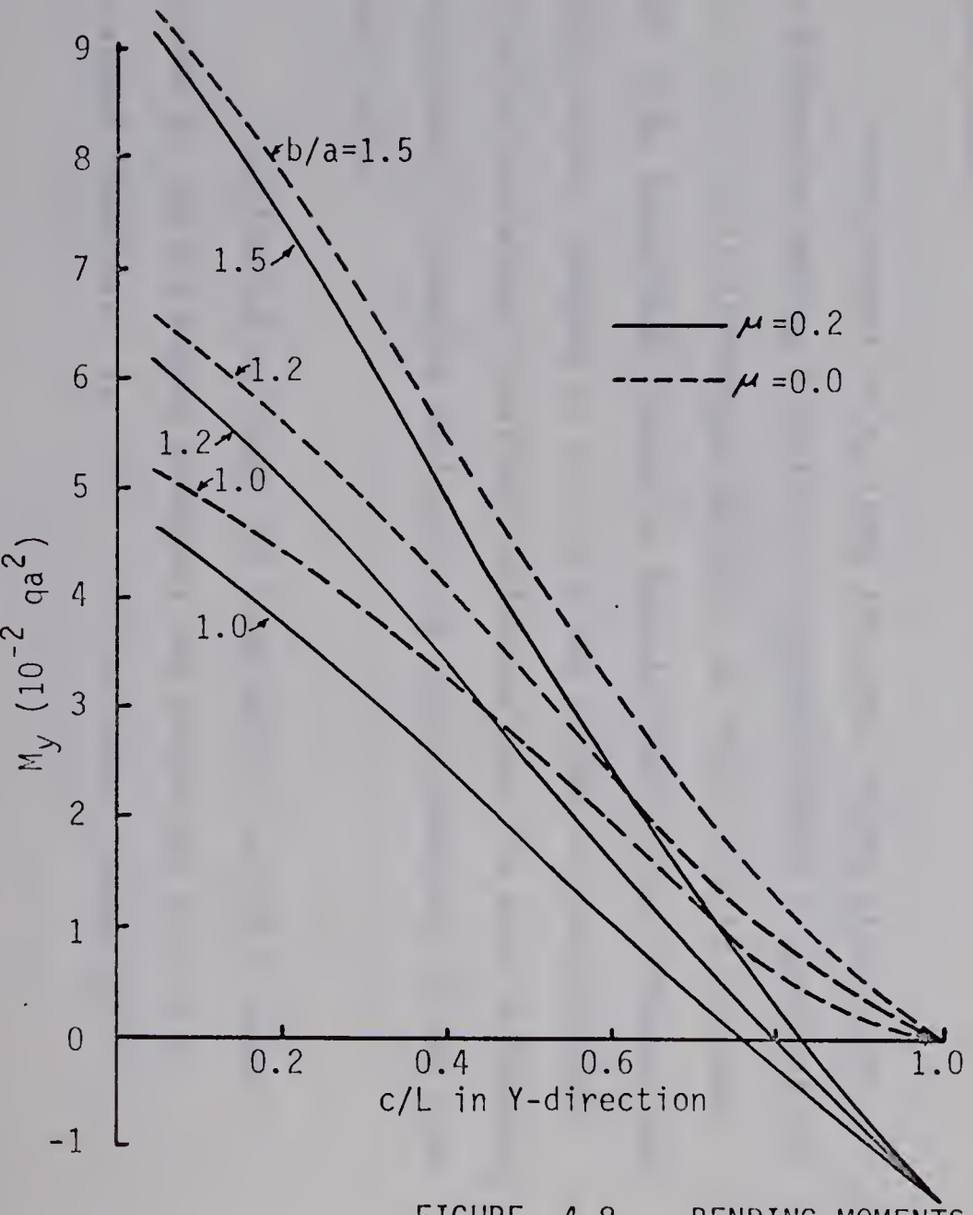


FIGURE 4.7 BENDING MOMENTS AT THE POINT ON THE X-AXIS MIDWAY BETWEEN COLUMNS
UNIFORM LOAD; b/a=1.0,1.2,1.5; μ =0.0,0.2





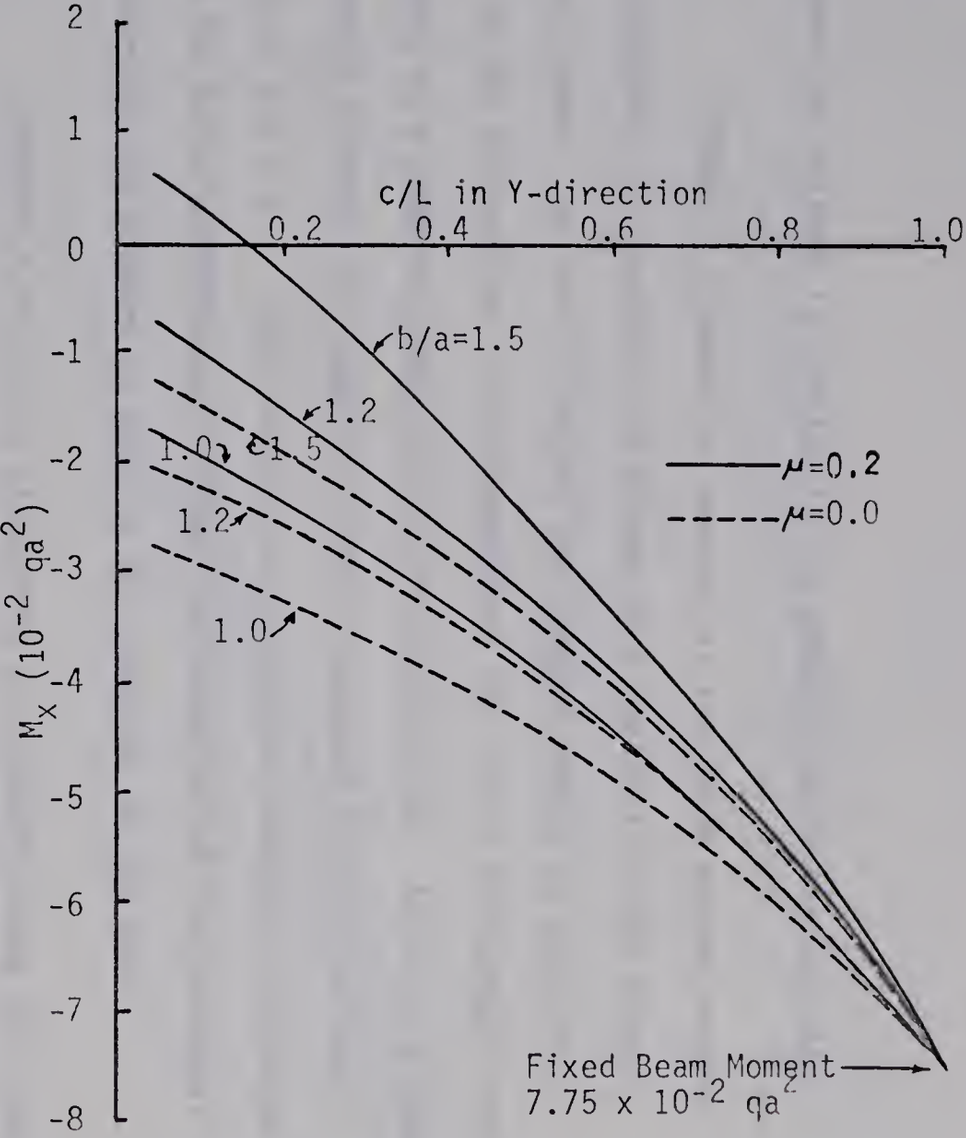
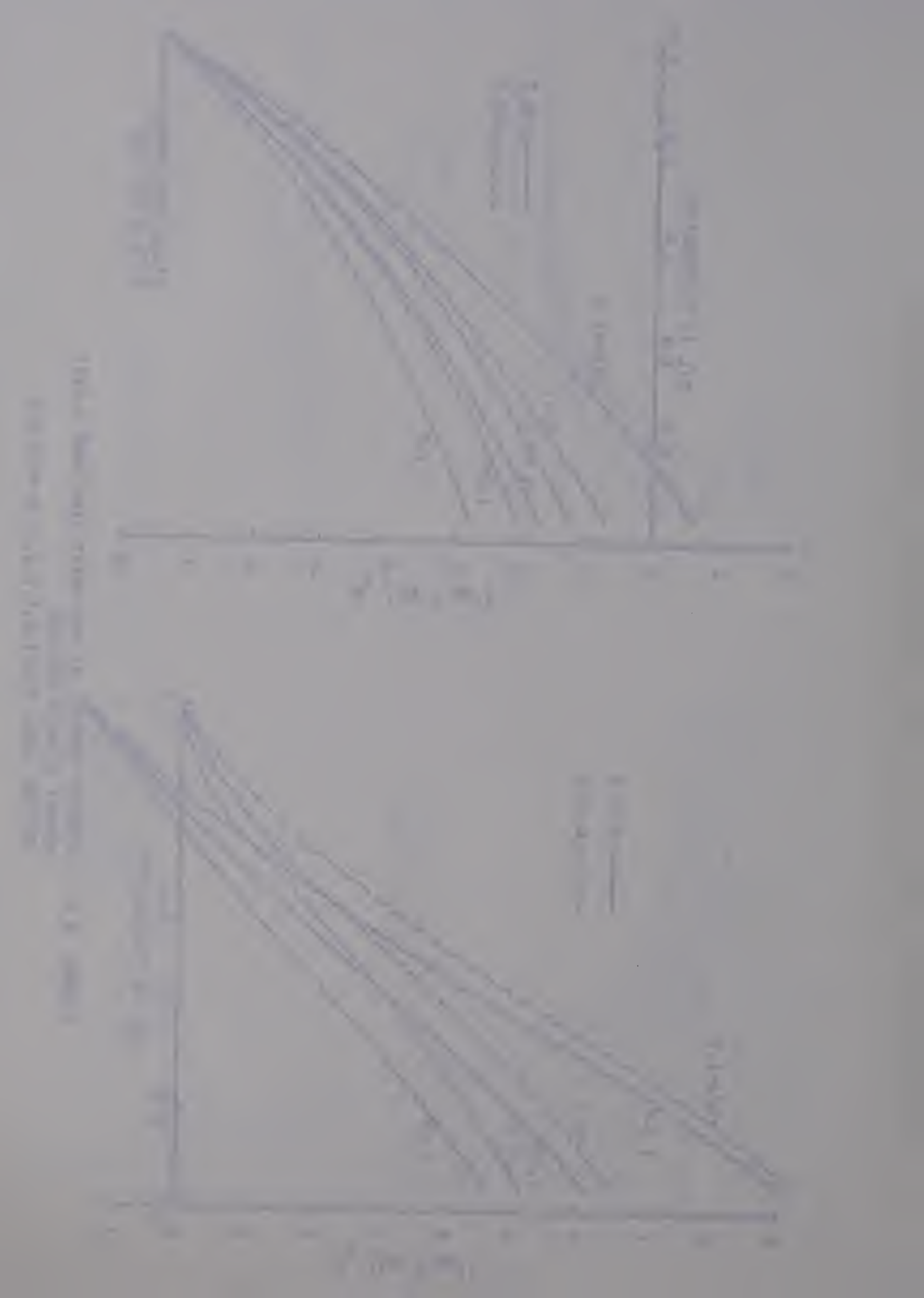


FIGURE 4.8 BENDING MOMENTS AT THE POINT a/40 FROM Y-AXIS
MIDWAY BETWEEN COLUMNS
UNIFORM LOAD; b/a=1.0,1.2,1.5; μ =0.0,0.2



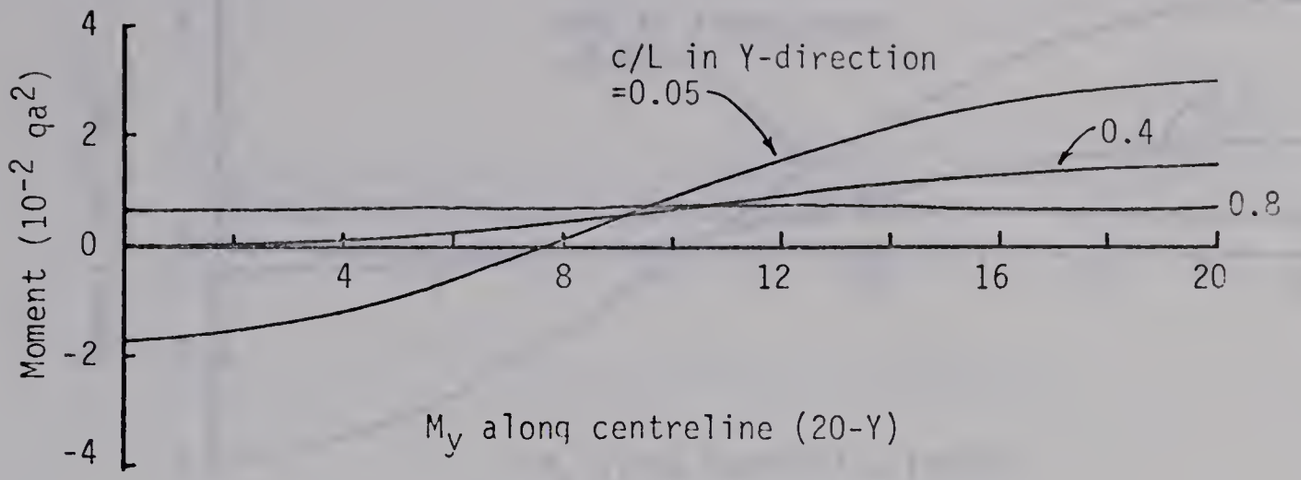
between the columns is similar to the behavior of a one-way slab.

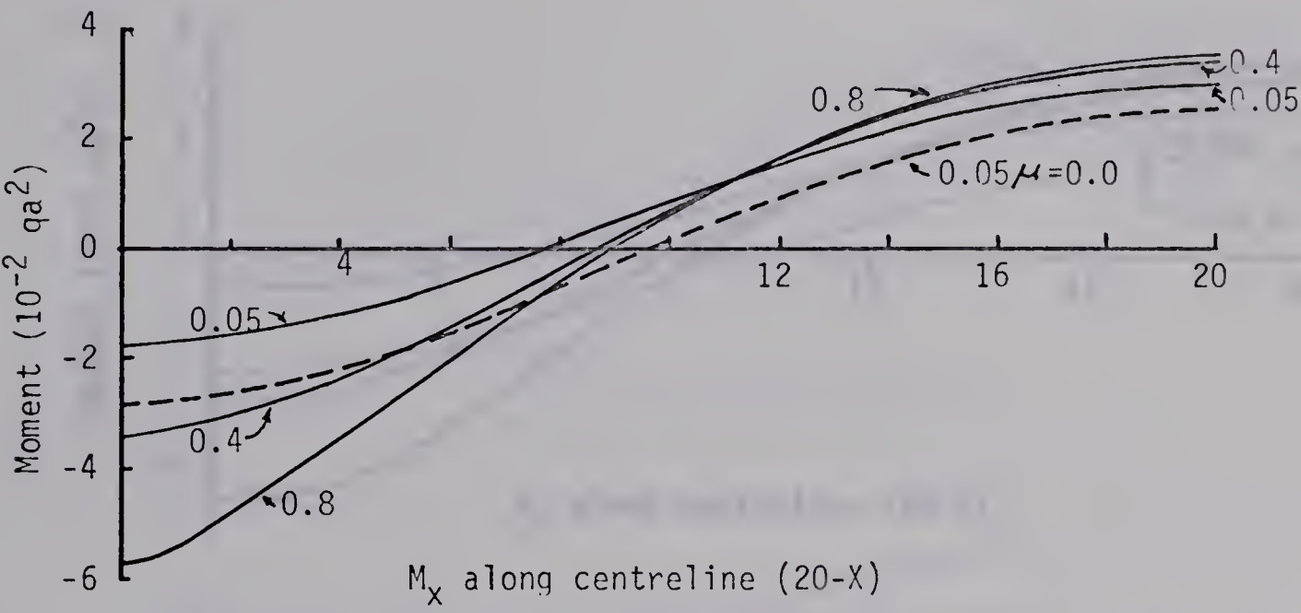
FIGURE 4.8 shows the moments at the point midway between the columns, in line with the column face parallel to the y-axis. In all cases, for low c/L ratios the largest moment is positive and in the y-direction. At intermediate c/L ratios the largest moment changes to a negative sense and in the x-direction. As the c/L ratio increases further this negative moment increases to its limiting value at c/L = 1.0. When applied to reinforced concrete structures this has major significance in that the direction of the principal reinforcing has changed as well as its position in the slab.

FIGURES 4.9 to 4.11 illustrate the moments along both panel centre lines and along the line in the x-direction defined by the column corner.

With respect to m_y along the panel centre line, 20-Y, the difference between positive and negative moment is greatest for c/L = 0.05. This difference decreases as the c/L ratio increases. There is no significant change in behavior due to the different slab aspect ratios. However at c/L = 0.8 the curves for the different b/a ratios are almost identical indicating that the slab is behaving predominantly in one-way action and is thus independent of the slab aspect ratio.

Considering m_x along the slab centre line 20-X, again for each of the b/a ratios examined, the curves for c/L = 0.8 are almost identical. For c/L = 0.05 the moment curves for





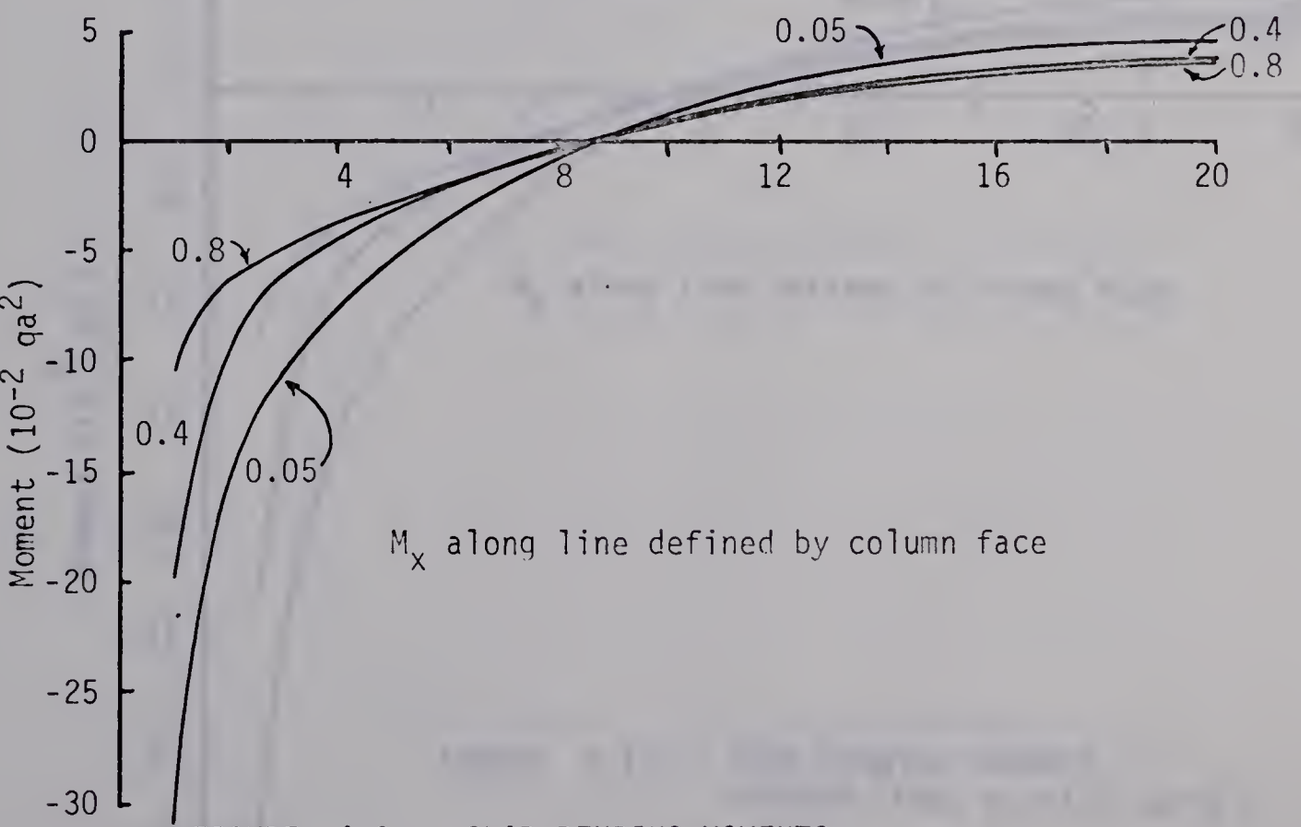
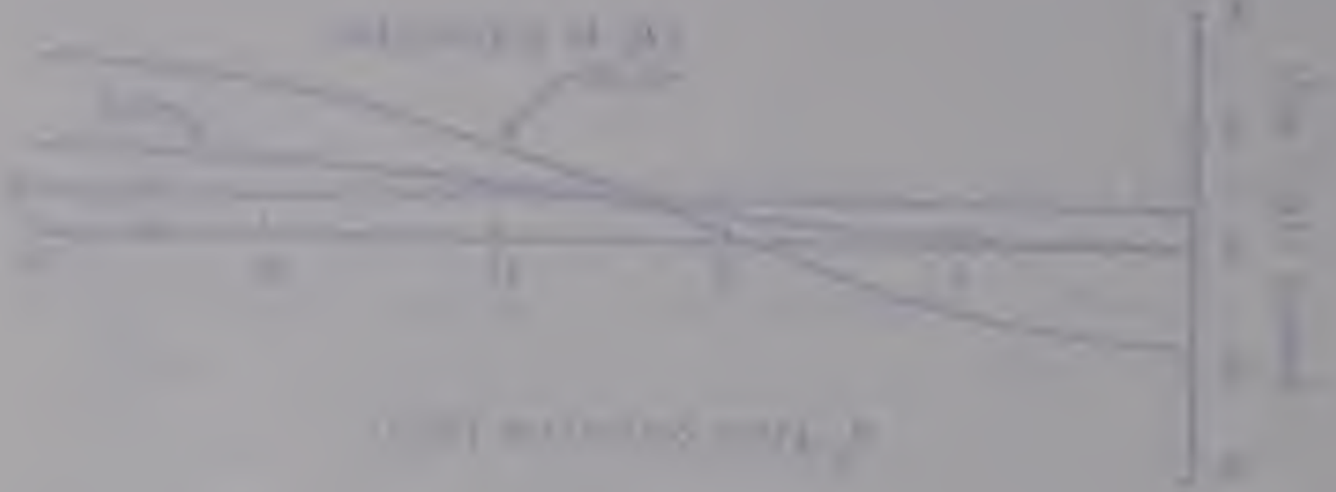
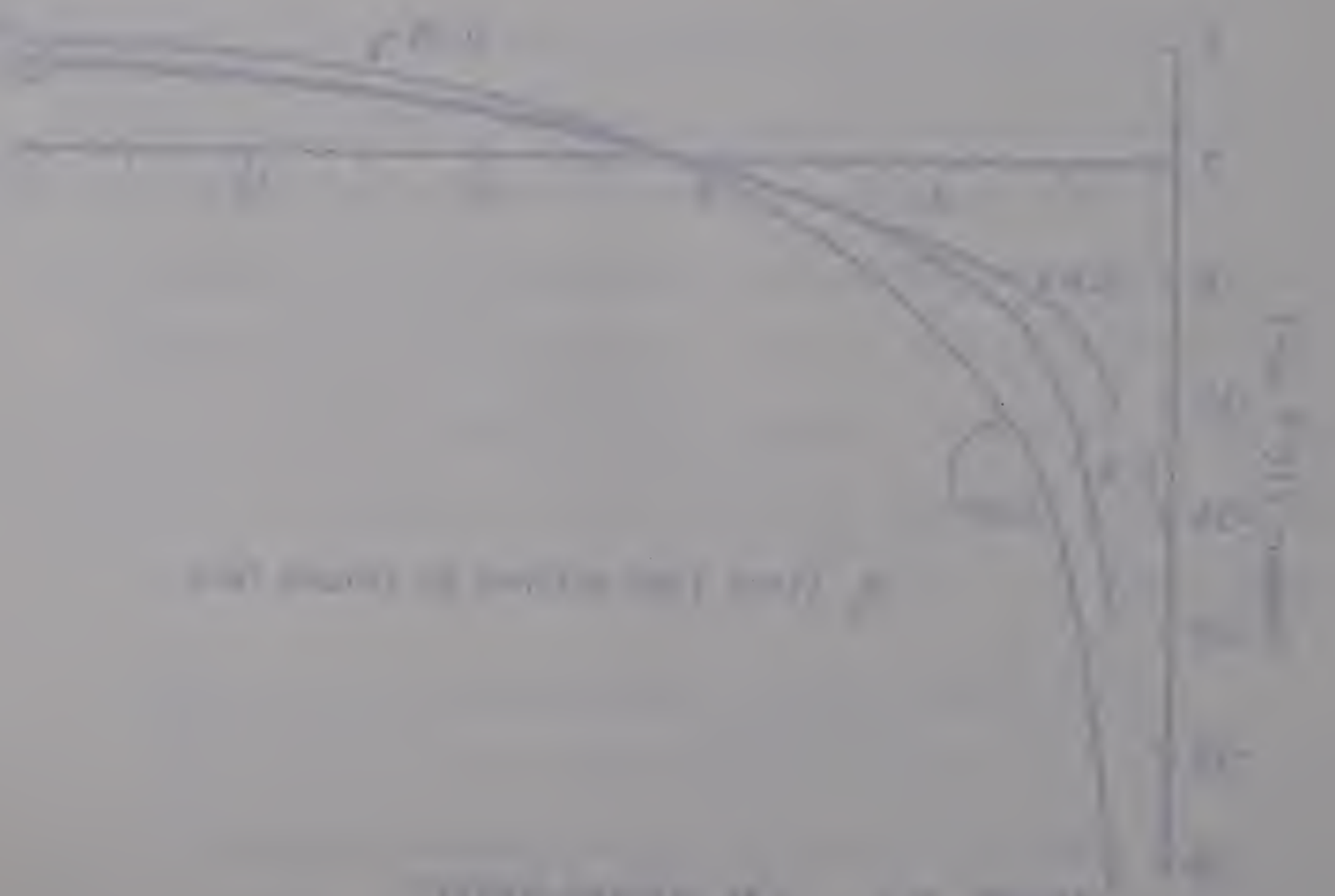


FIGURE 4.9 SLAB BENDING MOMENTS
UNIFORM LOAD; b/a=1.0; μ =0.2







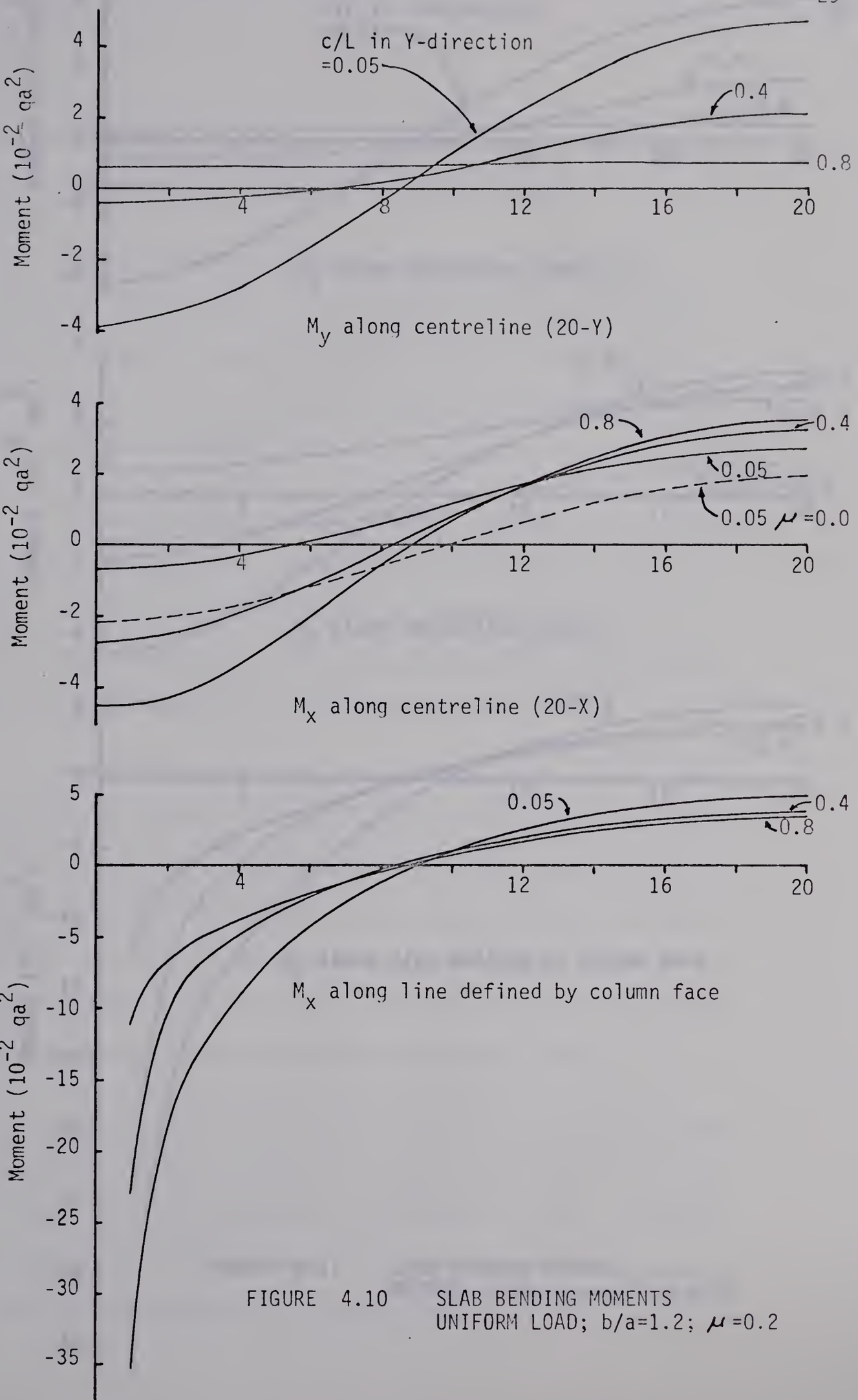
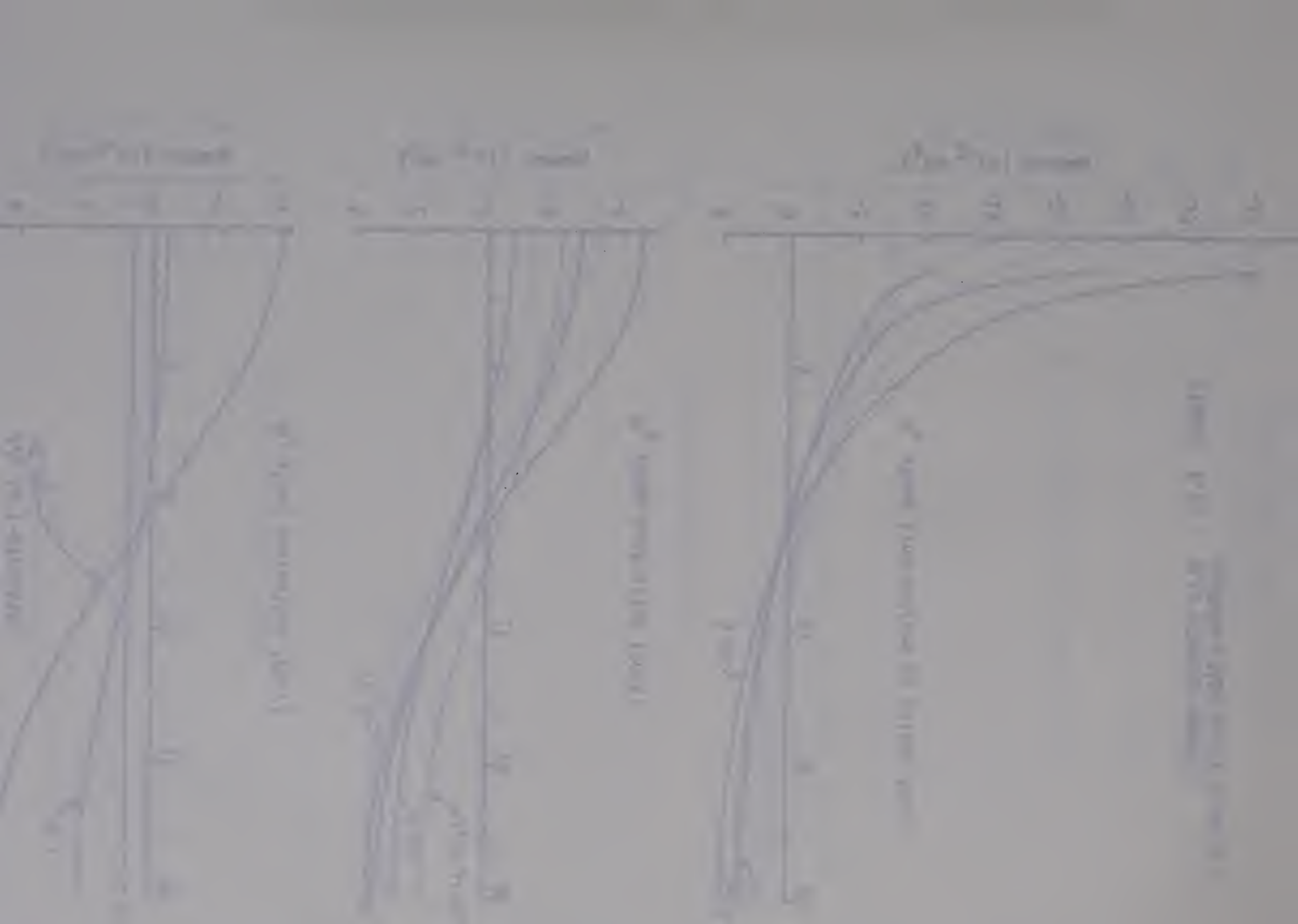


FIGURE 4.10 SLAB BENDING MOMENTS
UNIFORM LOAD; b/a=1.2; μ =0.2



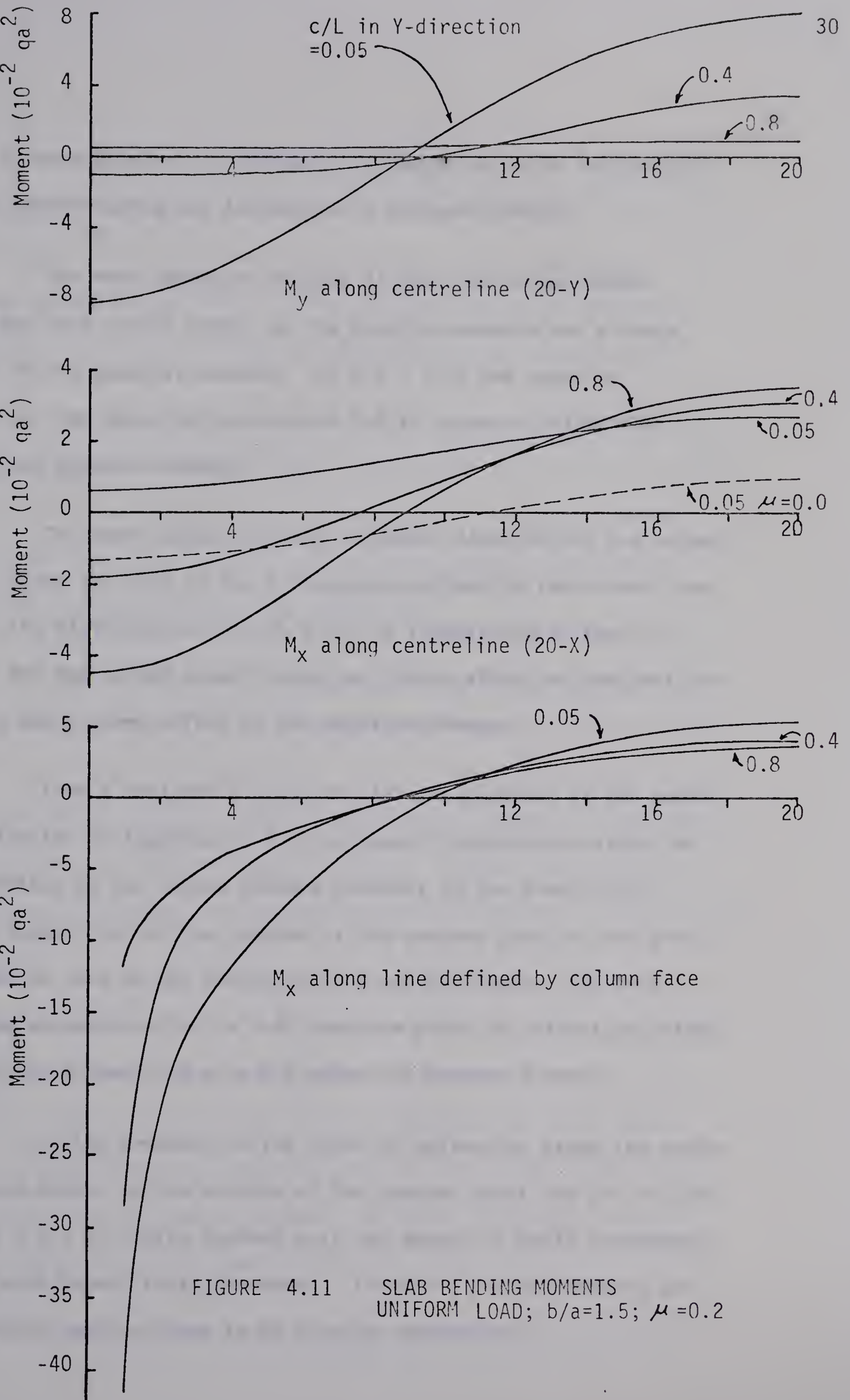
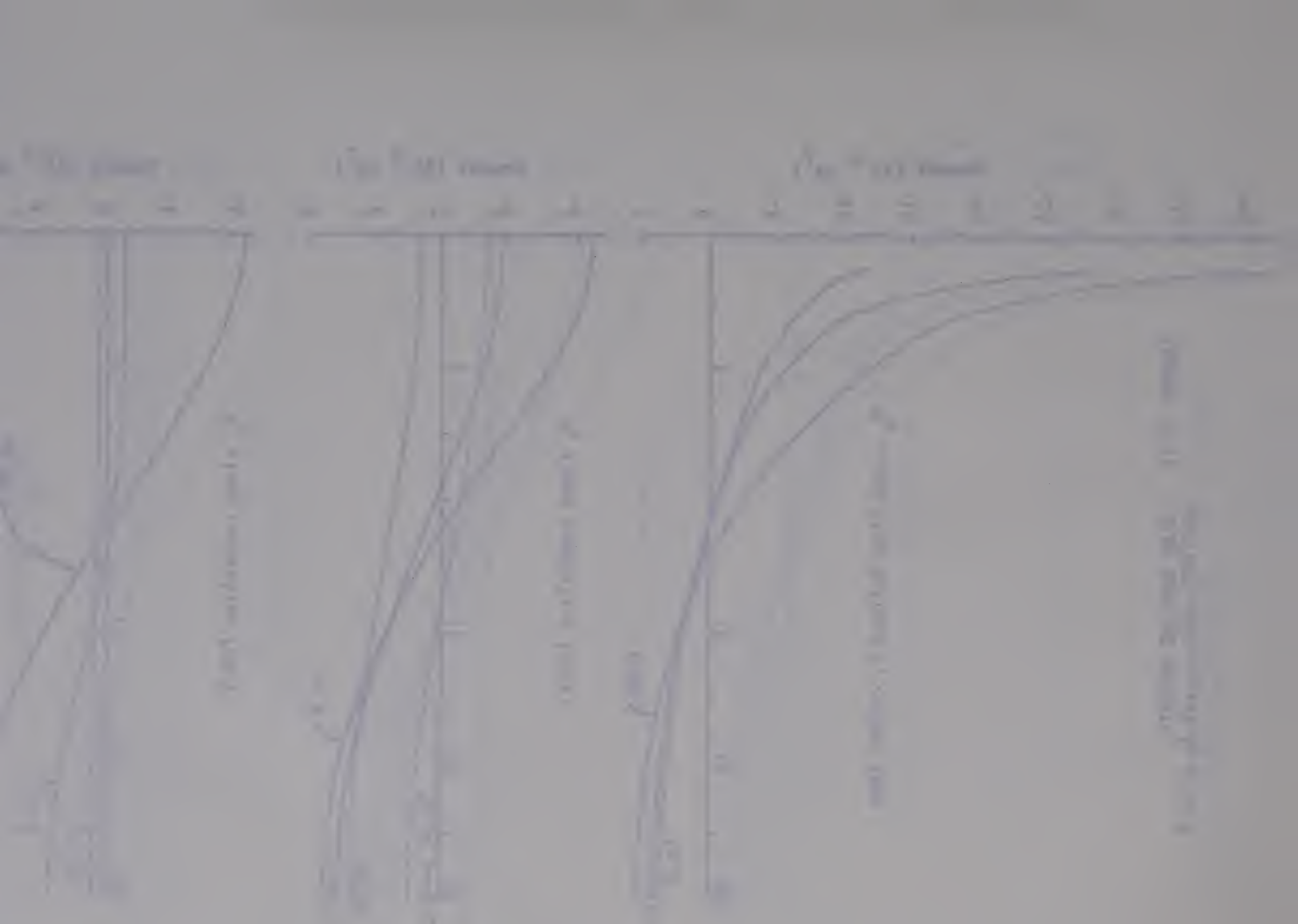


FIGURE 4.11 SLAB BENDING MOMENTS
UNIFORM LOAD; b/a=1.5; μ =0.2



$\mu = 0.0$ are also shown to indicate the degree to which the position of the moment curves are influenced by Poisson's ratio.

The most important feature is that the column aspect ratio has very little effect on the positive moments and a large effect on the negative moments. At c/L = 0.05 the negative moment is very small while at c/L = 0.8 it is about 75% of the fixed beam negative moment.

The same characteristics of moment distribution are evident for m_x along the line in the x-direction defined by the column face. Again, the distribution for c/L = 0.8 is insensitive to the b/a ratio, and the column aspect ratio has little effect on the positive moments and a great effect on the negative moments.

From a designer's point of view the position of the point of inflection is important. For the moment distribution along the line defined by the column corners parallel to the x-axis this always occurs just to the outside of the quarter point of the slab. The same is true of the distribution along the centre line 20-X with the exception of c/L = 0.05 when the point of inflection shifts farther out primarily due to the effect of Poisson's ratio.

In the y-direction the point of inflection along the centre line also occurs to the outside of the quarter point for c/L = 0.05. For c/L = 0.4 it shifts further out; the amount of shift decreasing as the slab aspect ratio increases. For c/L = 0.8 the moments are all positive and so there is no point of inflection.

FIGURES 4.12 to 4.14 show the distribution of m_x across lines 20-Y, 10-Y, and 1-Y for c/L = 0.05, 0.4 and 0.8. There are no significant changes that can be attributed to the slab aspect ratio. However as was evident for the moments along lines in the slab, the behavior at c/L = 0.8 is relatively independent of the b/a ratio.

Two features are noteworthy with respect to the positive values of m_x across 20-Y and 10-Y. The maximum moment always occurs near the point on line with the column corner. Also only the m_x values for c/L = 0.05 exhibit a significant change in magnitude between the column line and the centre line.

For the negative values of m_x across 1-Y the maximum ordinate always occurs at the column corner. Along the face of the column m_x decreases substantially. For all b/a ratios m_x across the column face for c/L = 0.4 and 0.8 is almost the same and almost equal to the maximum fixed beam negative moment. Towards the middle of the slab m_x decreases, the maximum decrease occurring for the smallest c/L ratio.

FIGURE 4.15 shows the distribution of m_y across the centre line 20-X. This distribution is similar for all three b/a ratios examined. For c/L = 0.05 and 0.4 the maximum value of m_y occurs at the column line and m_y decreases toward the centre of the slab. For c/L = 0.8 m_y increases slightly toward the centre of the slab due to the effect of Poisson's ratio.

FIGURES 4.16 to 4.19 show the maximum slab moments for

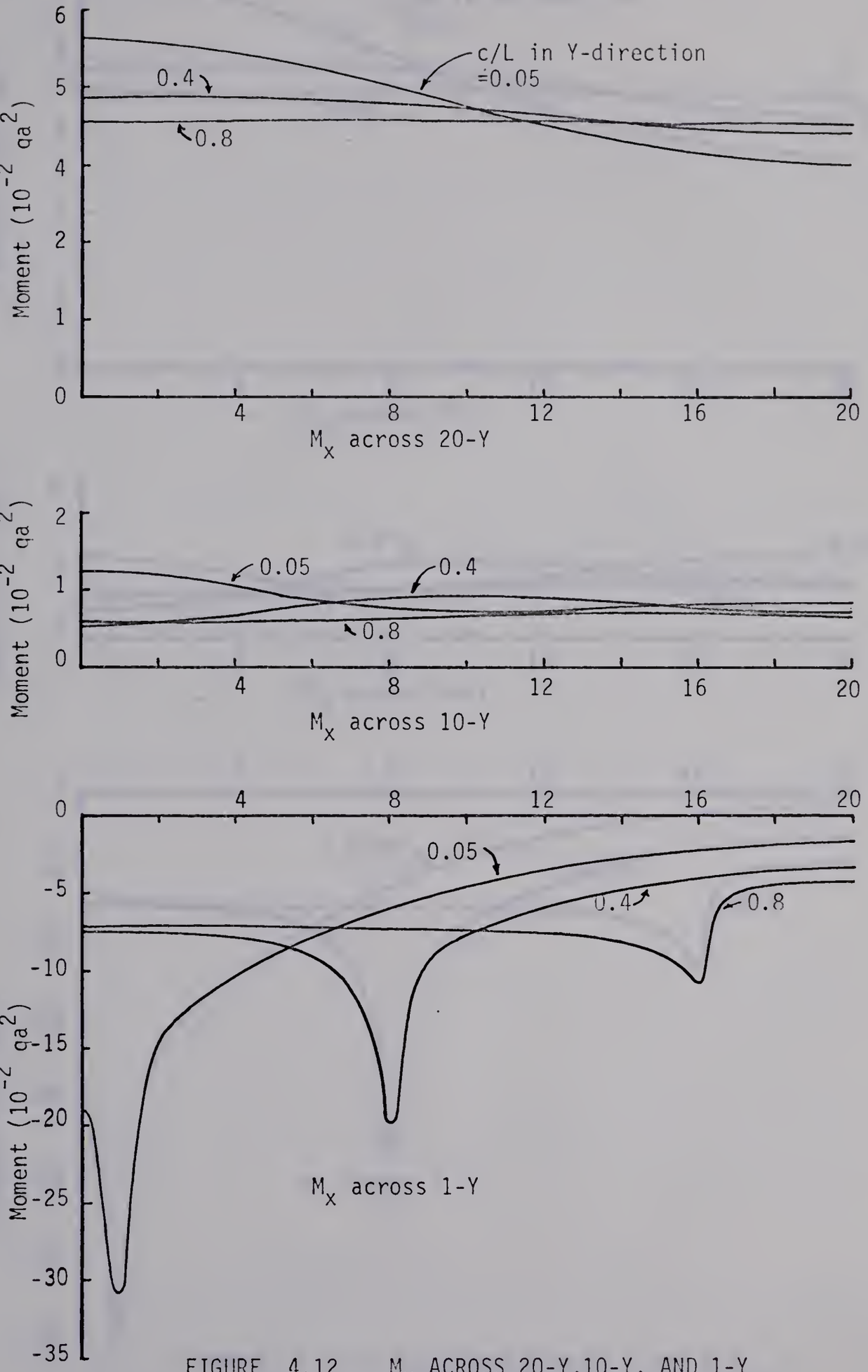
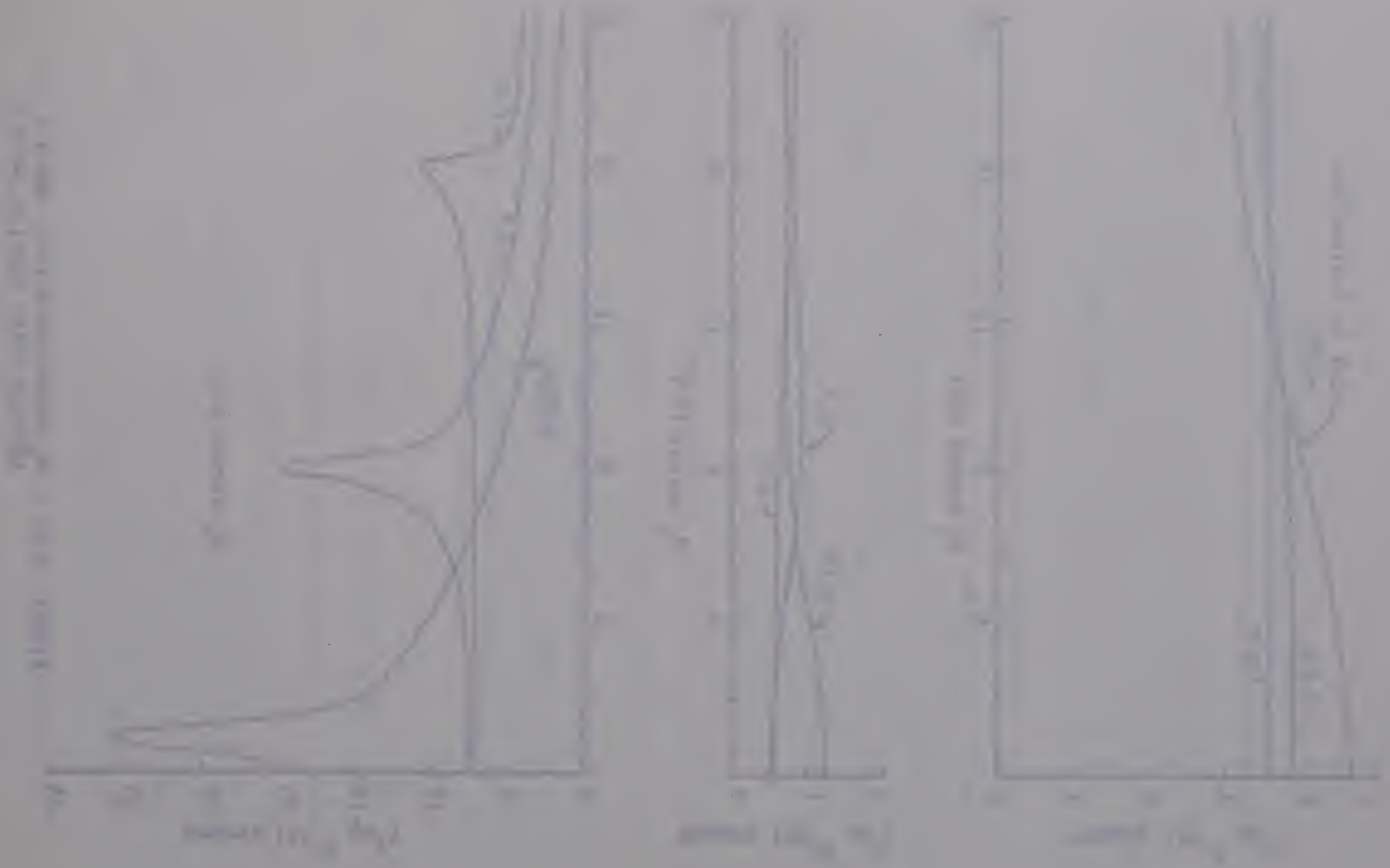
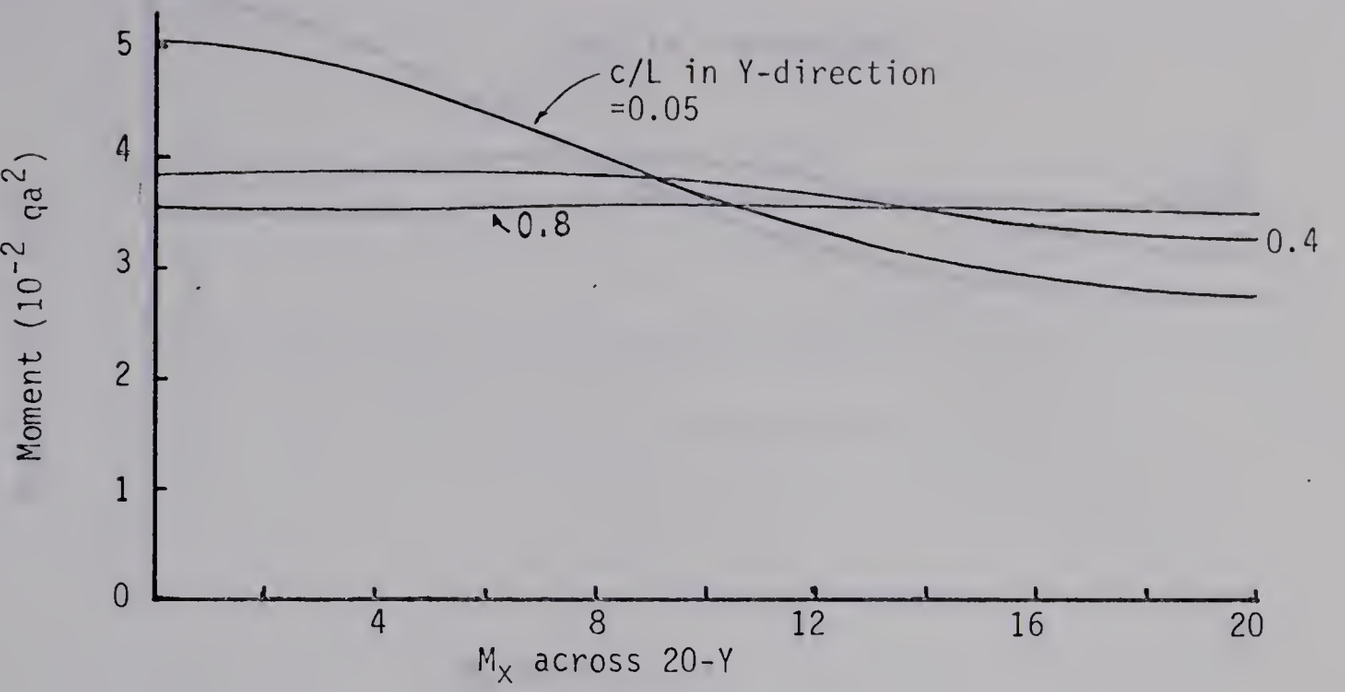
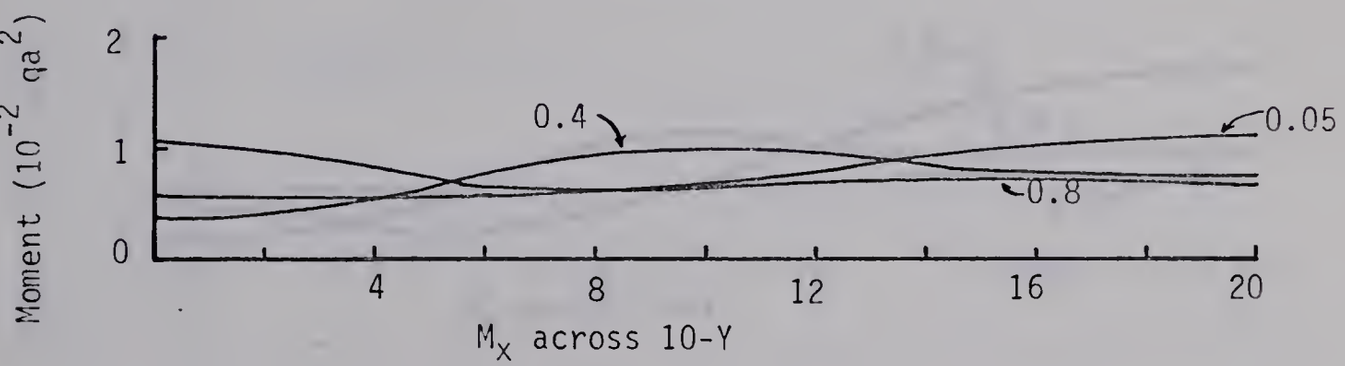


FIGURE 4.12 M_x ACROSS 20-Y, 10-Y, AND 1-Y
UNIFORM LOAD; b/a=1.0; μ =0.2







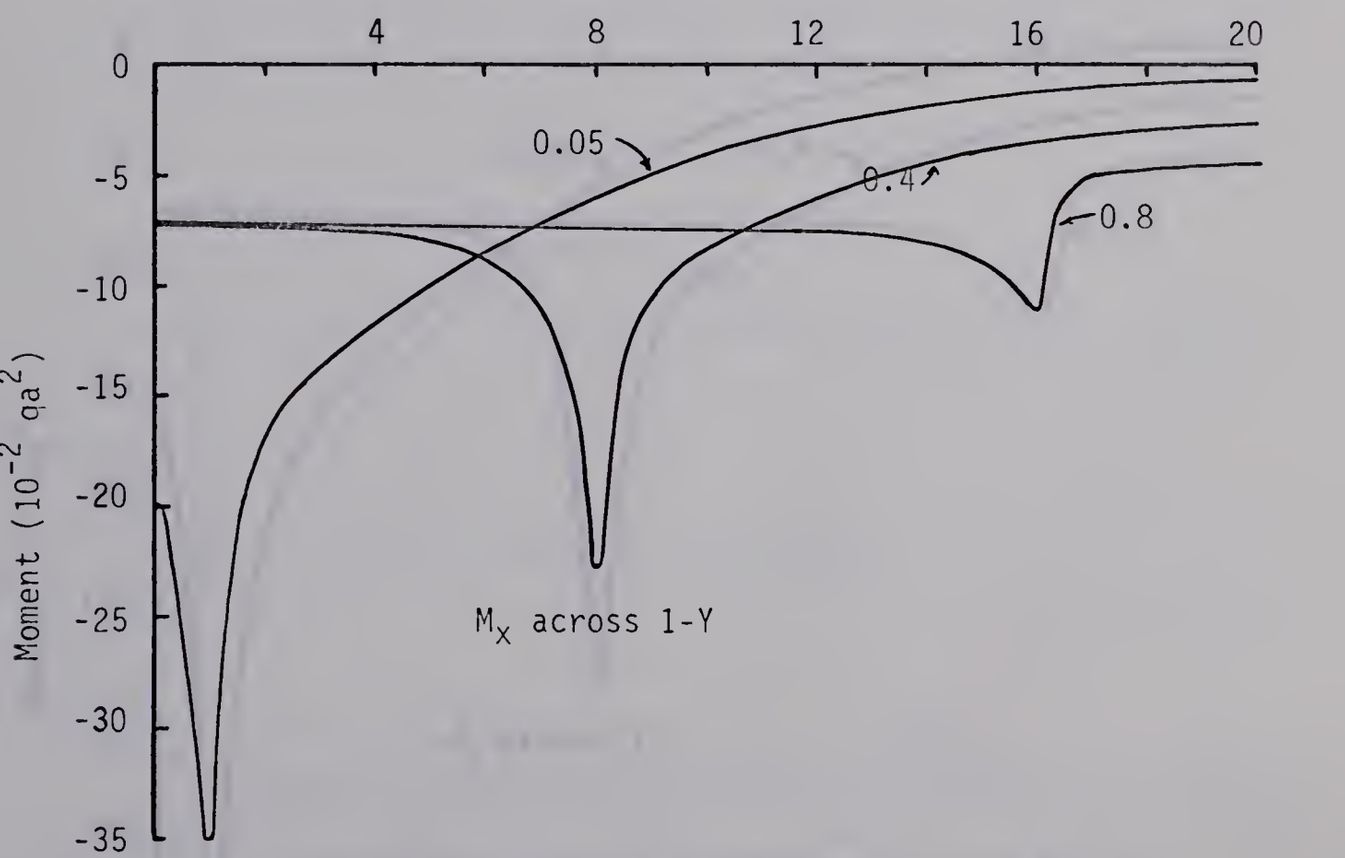
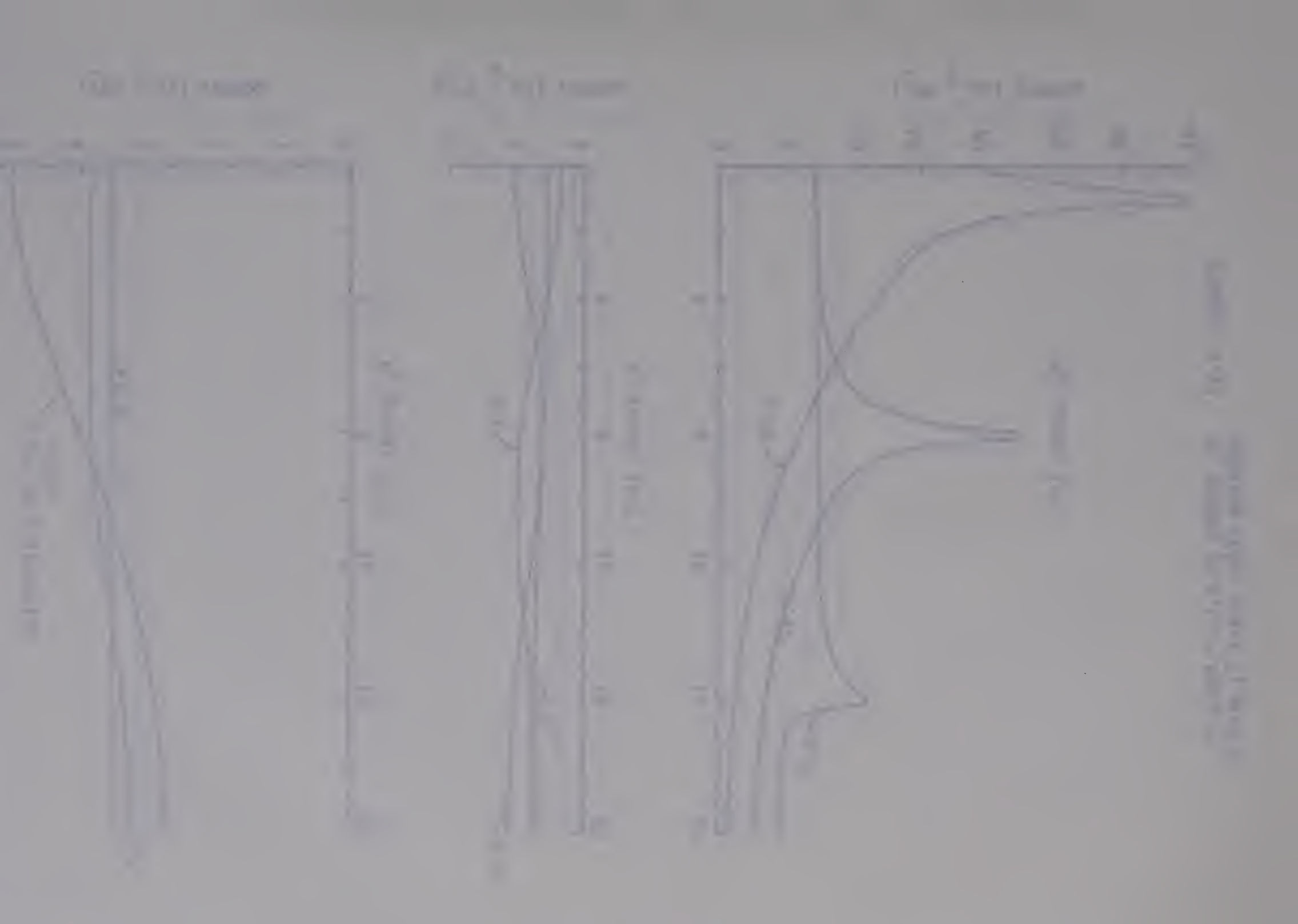
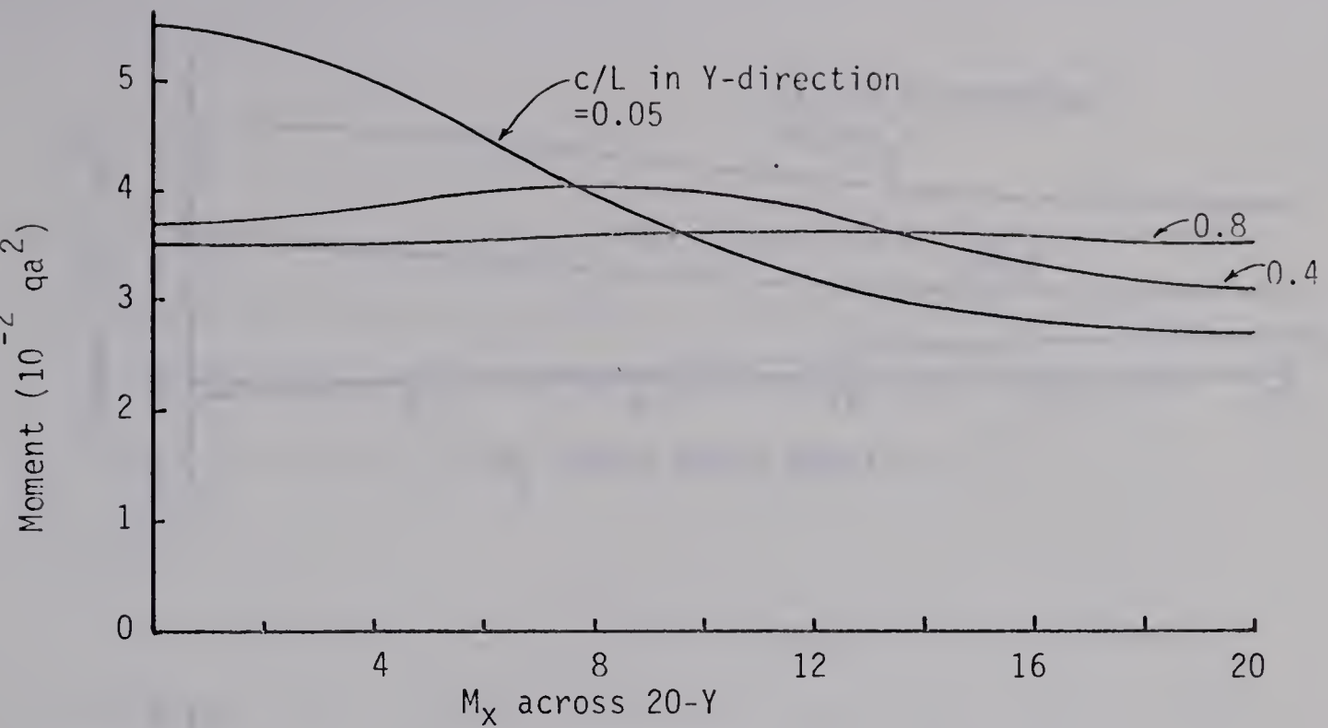
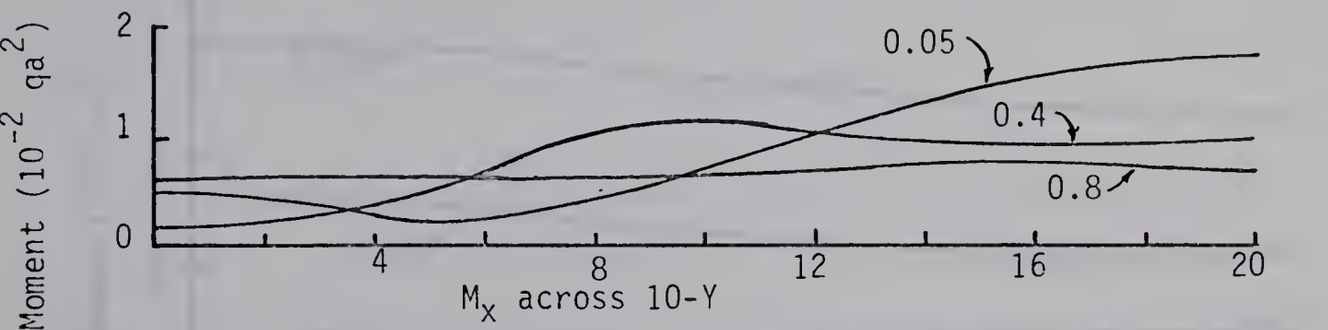


FIGURE 4.13 M_x ACROSS 20-Y,10-Y AND 1-Y
UNIFORM LOAD; b/a=1.2; μ =0.2







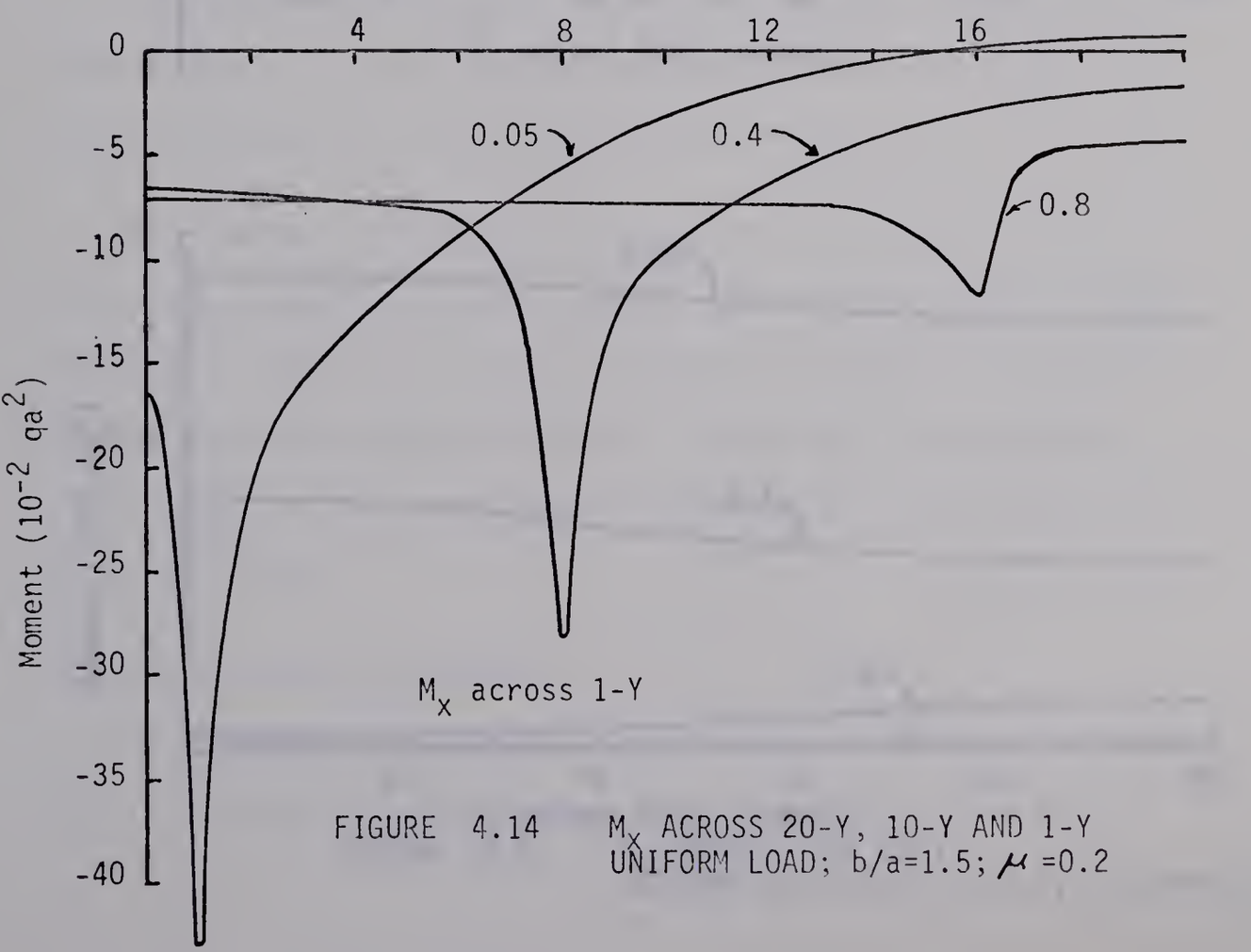
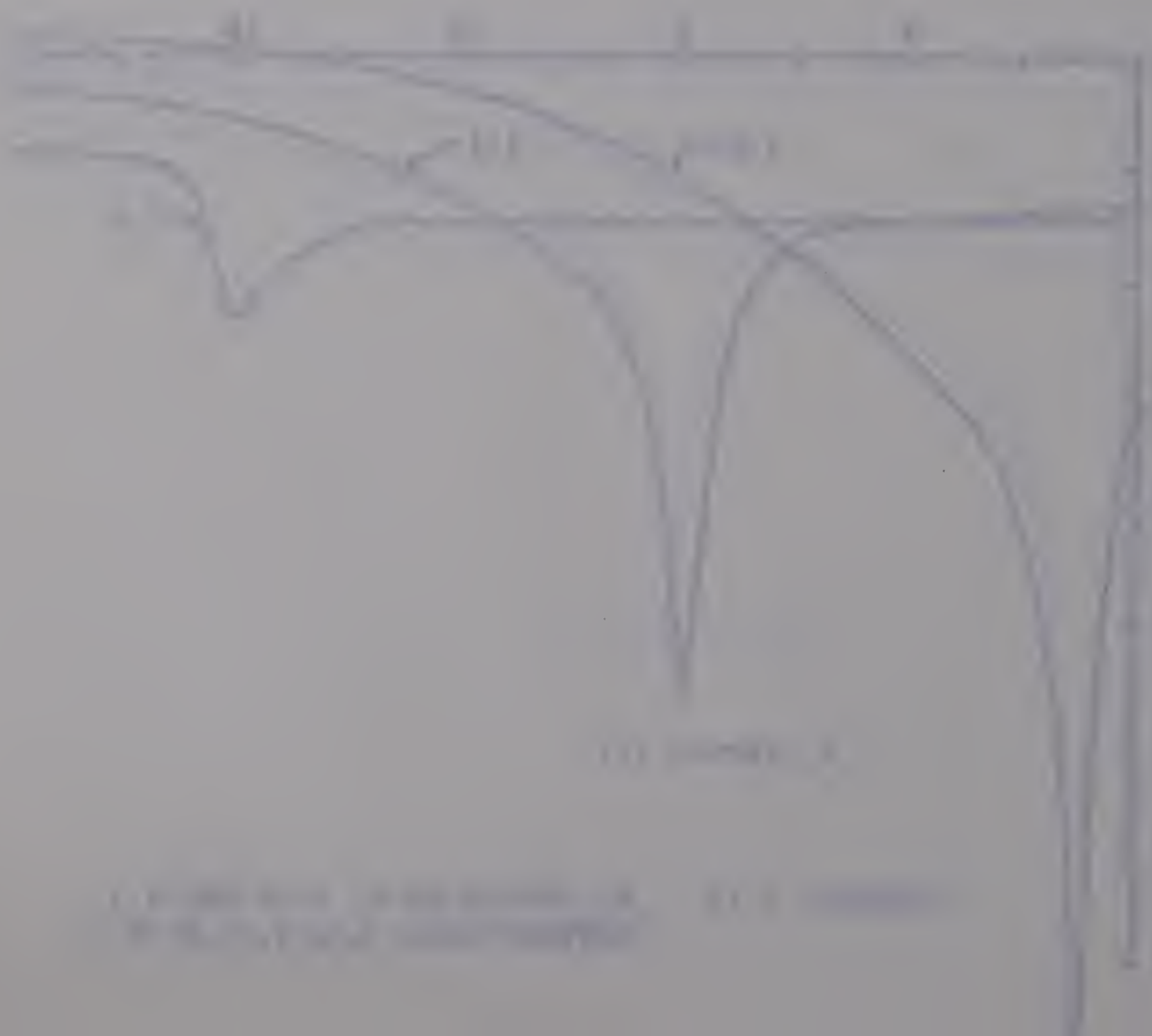
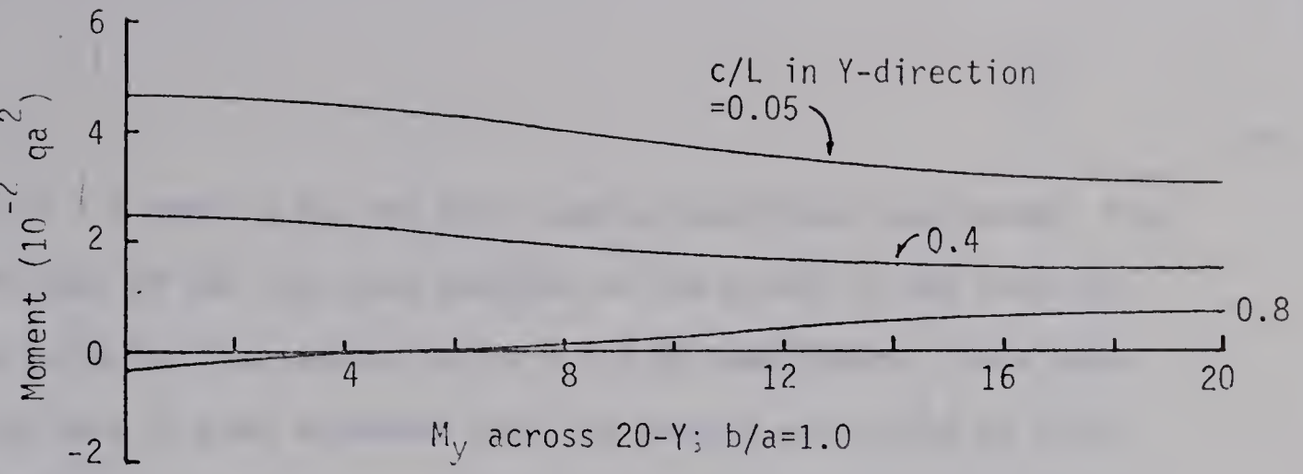


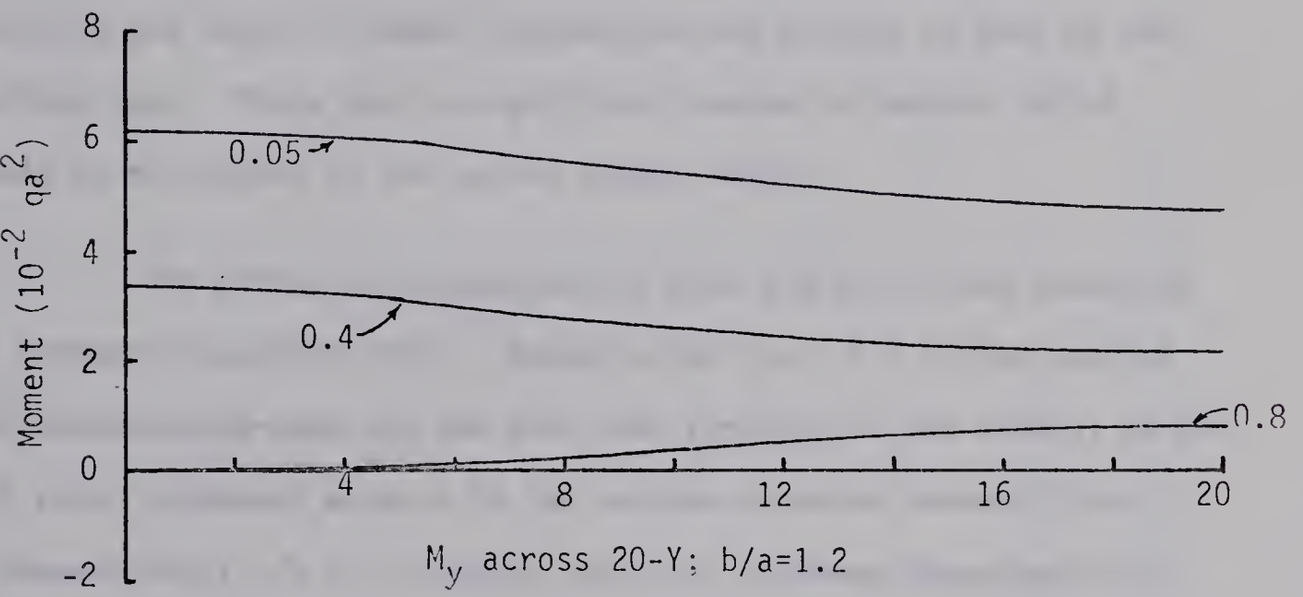
FIGURE 4.14 M_x ACROSS 20-Y, 10-Y AND 1-Y
UNIFORM LOAD; b/a=1.5; μ =0.2











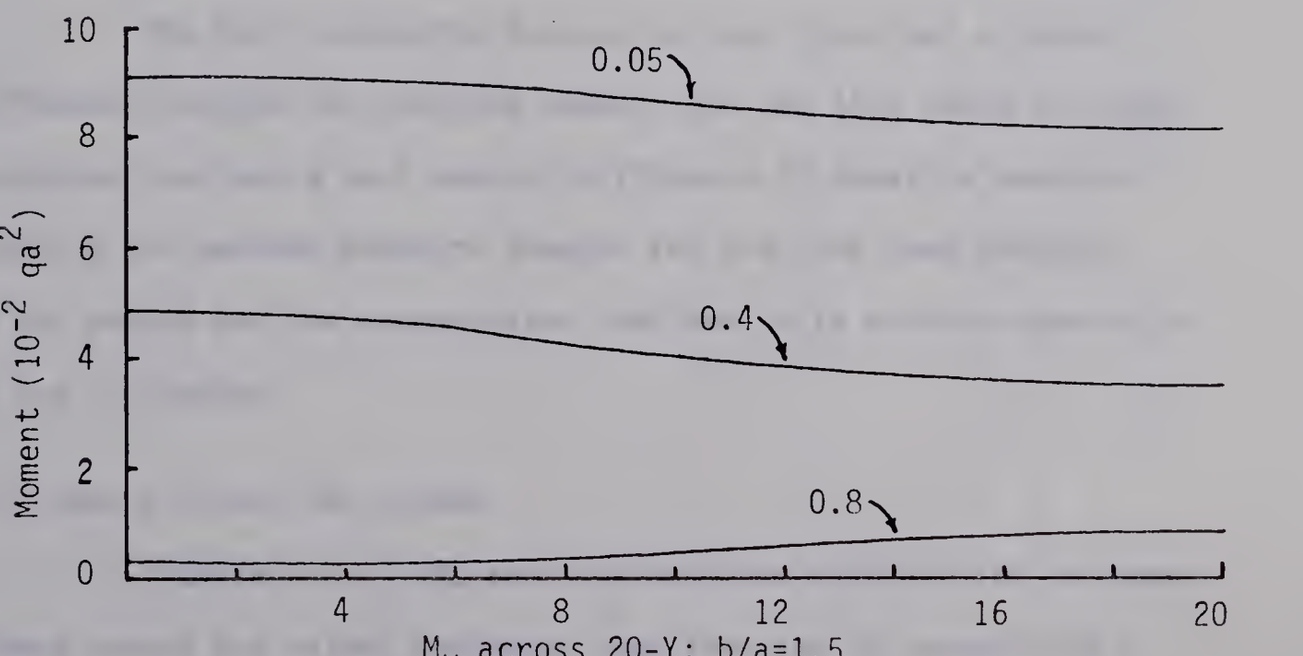
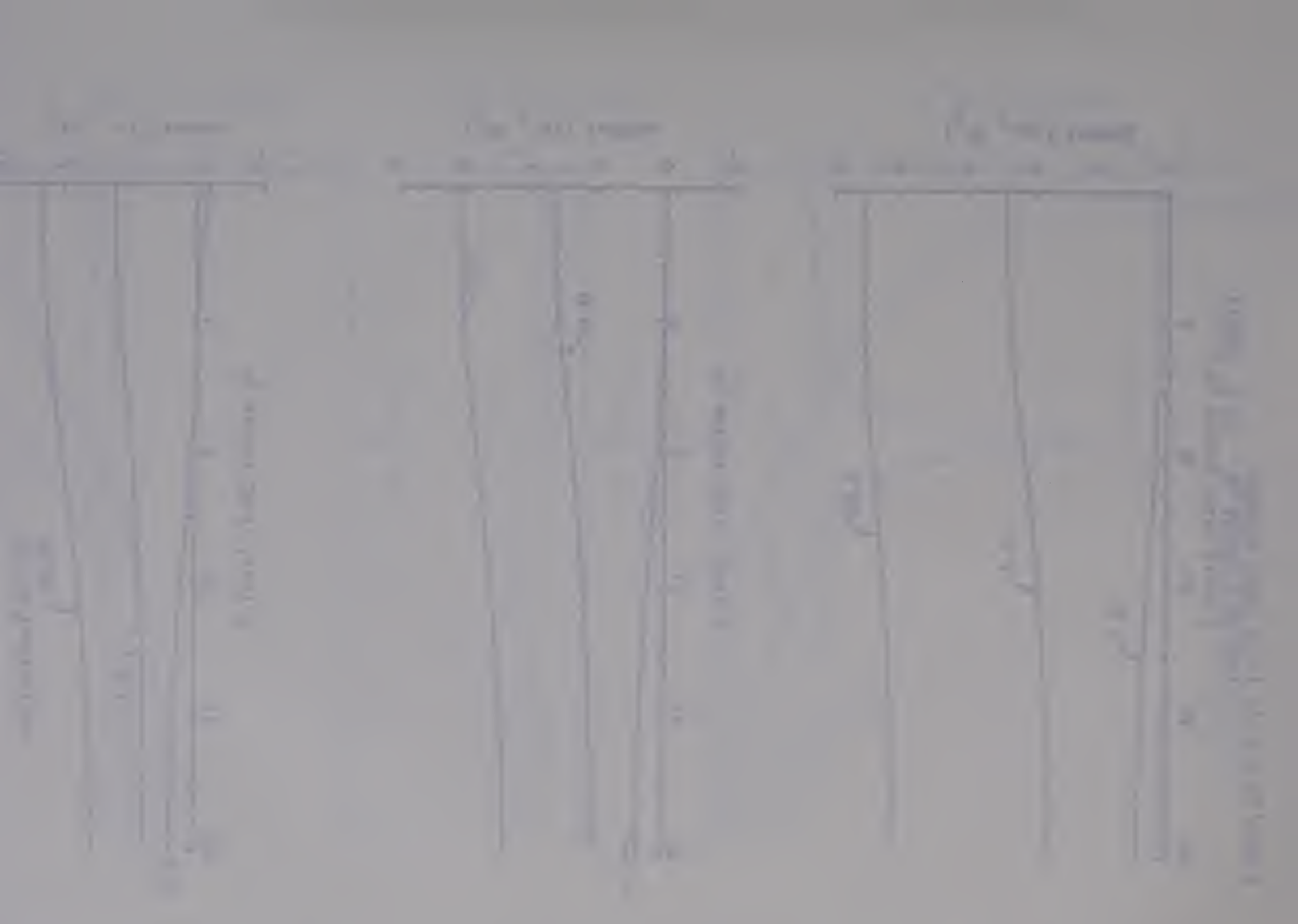


FIGURE 4.15 M_y ACROSS LINE 20-Y
UNIFORM LOAD; b/a=1.0,1.2,1.5; μ =0.2



b/a = 1.0 and 1.5 for the four loading conditions considered. For the case of the line load parallel to the y-axis it was possible to solve for the moments at c/L = 1.0 by beam theory. Once again they were in good agreement with the results calculated by finite differences.

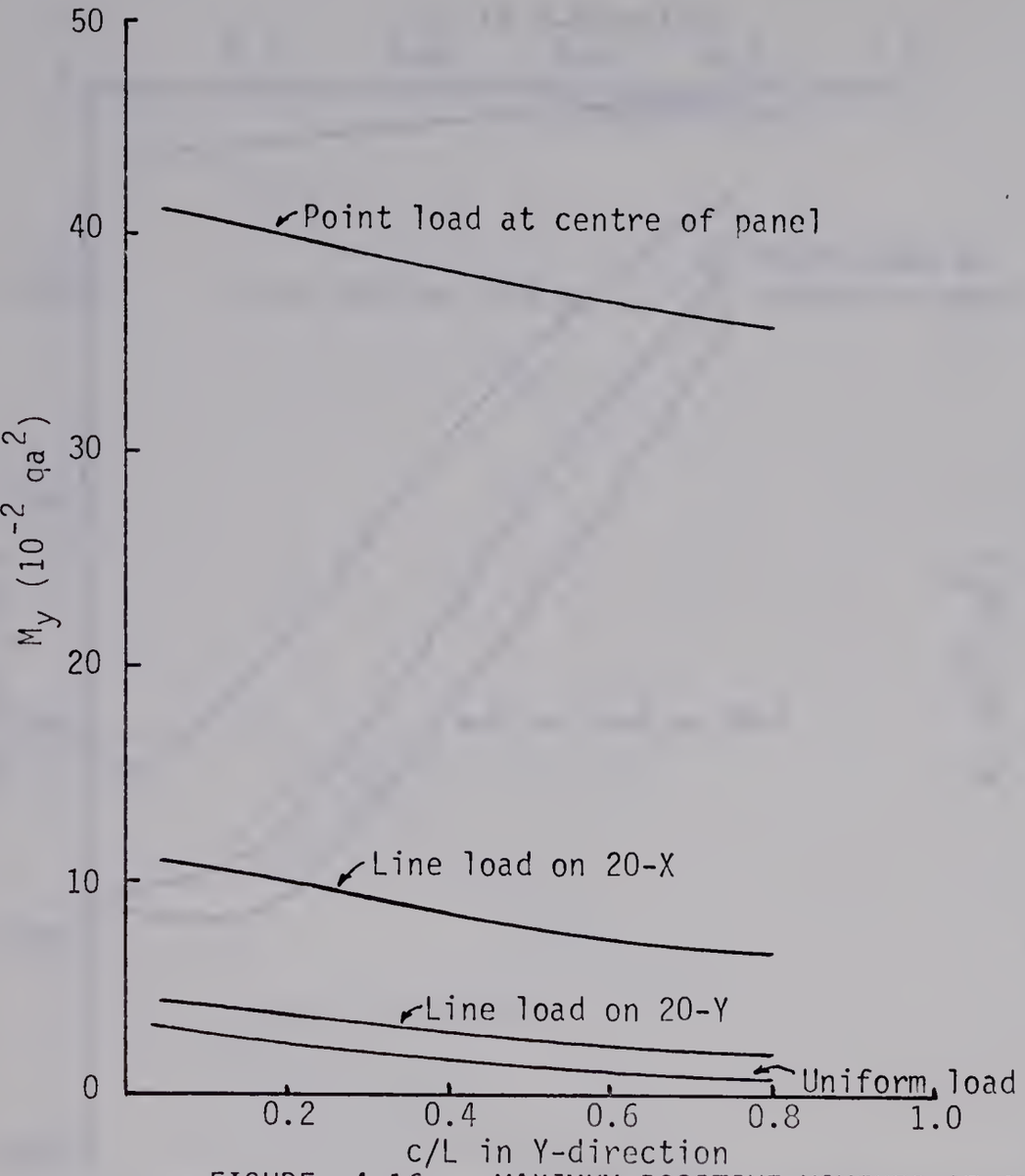
The behavior for the line loads and point loads for both positive and negative moment intensities was similar to that of the uniform load. There was no significant change in behavior which could be attributed to the column aspect ratio.

The effect of increasing b/a from 1.0 to 1.5 was generally to increase magnitudes only. However, for b/a = 1.5 in the case of the concentrated load and the line load parallel to the x-axis, as the c/L ratio increased above 0.05 the maximum negative moments first increased until c/L = 0.2 before the rapid decrease characteristic of the uniform load to the limiting moment at c/L = 1.0.

The most noteworthy feature is that there was a large difference between the positive moments for the line loads and concentrated load and a much smaller difference in negative moments. Actually the maximum negative moments for the line load parallel to the x-axis and the concentrated load are quite similar especially at low c/L ratios.

4.4 Shears Around The Column

FIGURES 4.20, 4.21 and 4.22 show the distribution of shear stress around the column perimeter when the slab is loaded with a



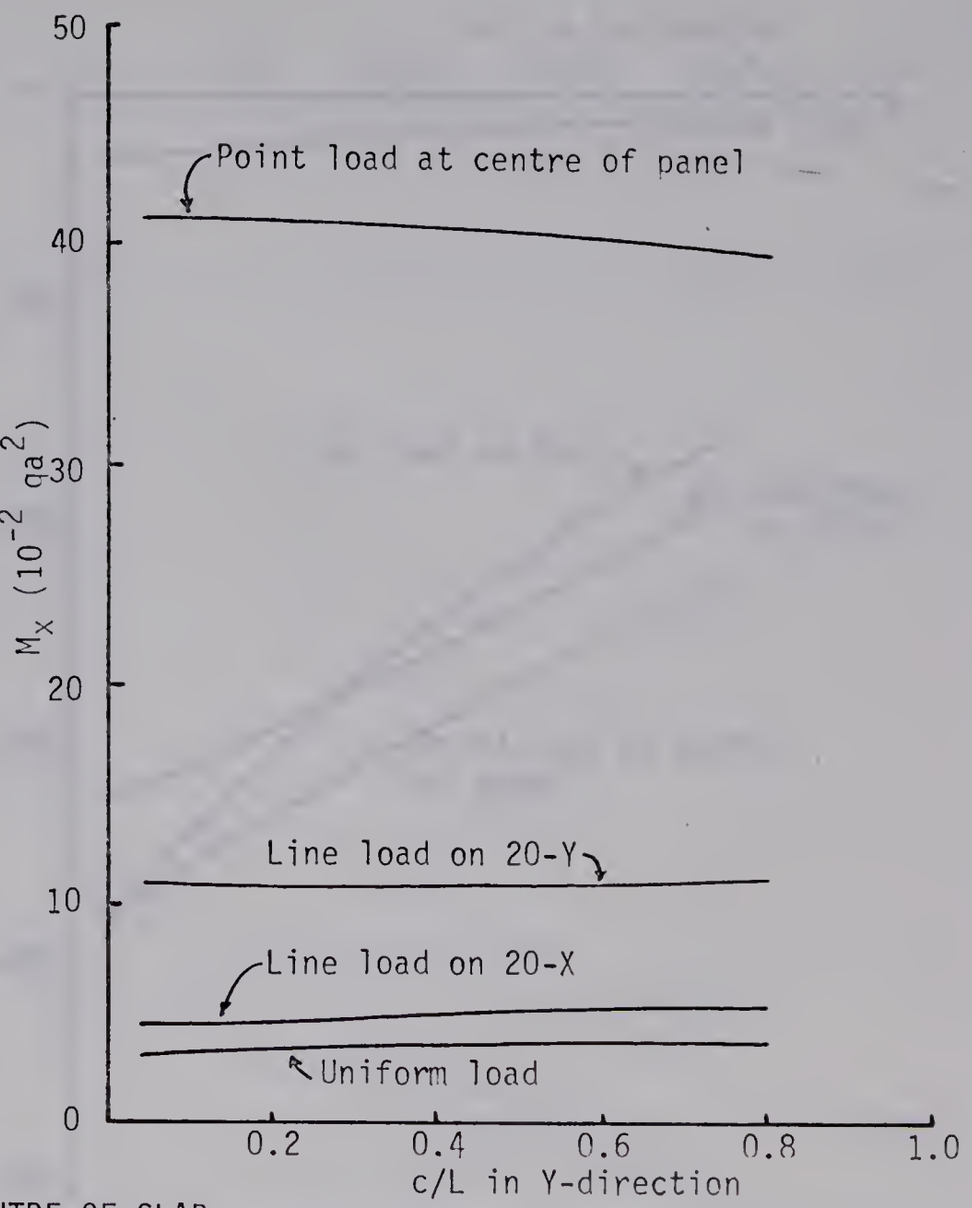
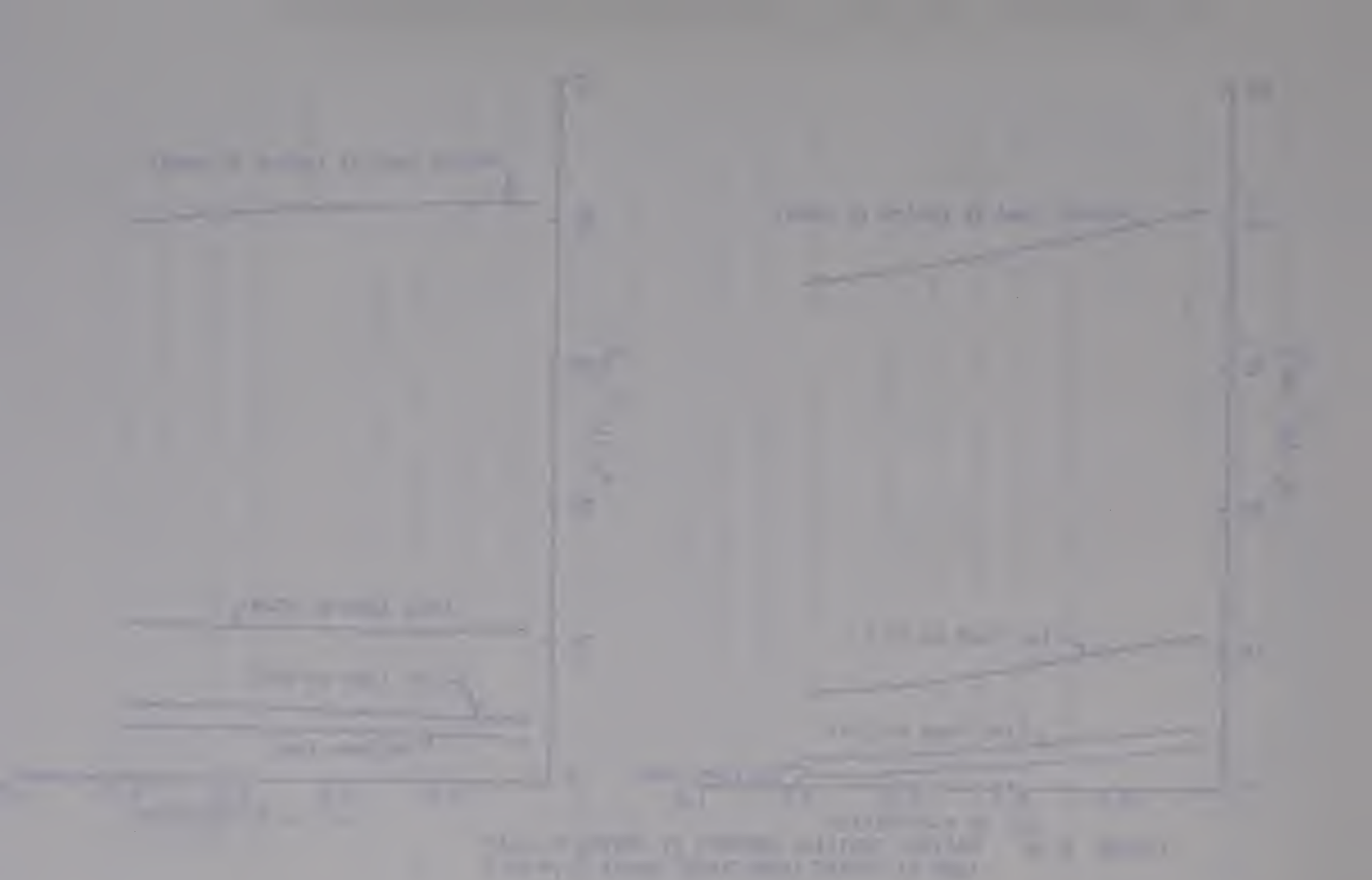


FIGURE 4.16 MAXIMUM POSITIVE MOMENTS AT CENTRE OF SLAB
FOUR DIFFERENT LOAD TYPES; b/a=1.0; μ =0.2



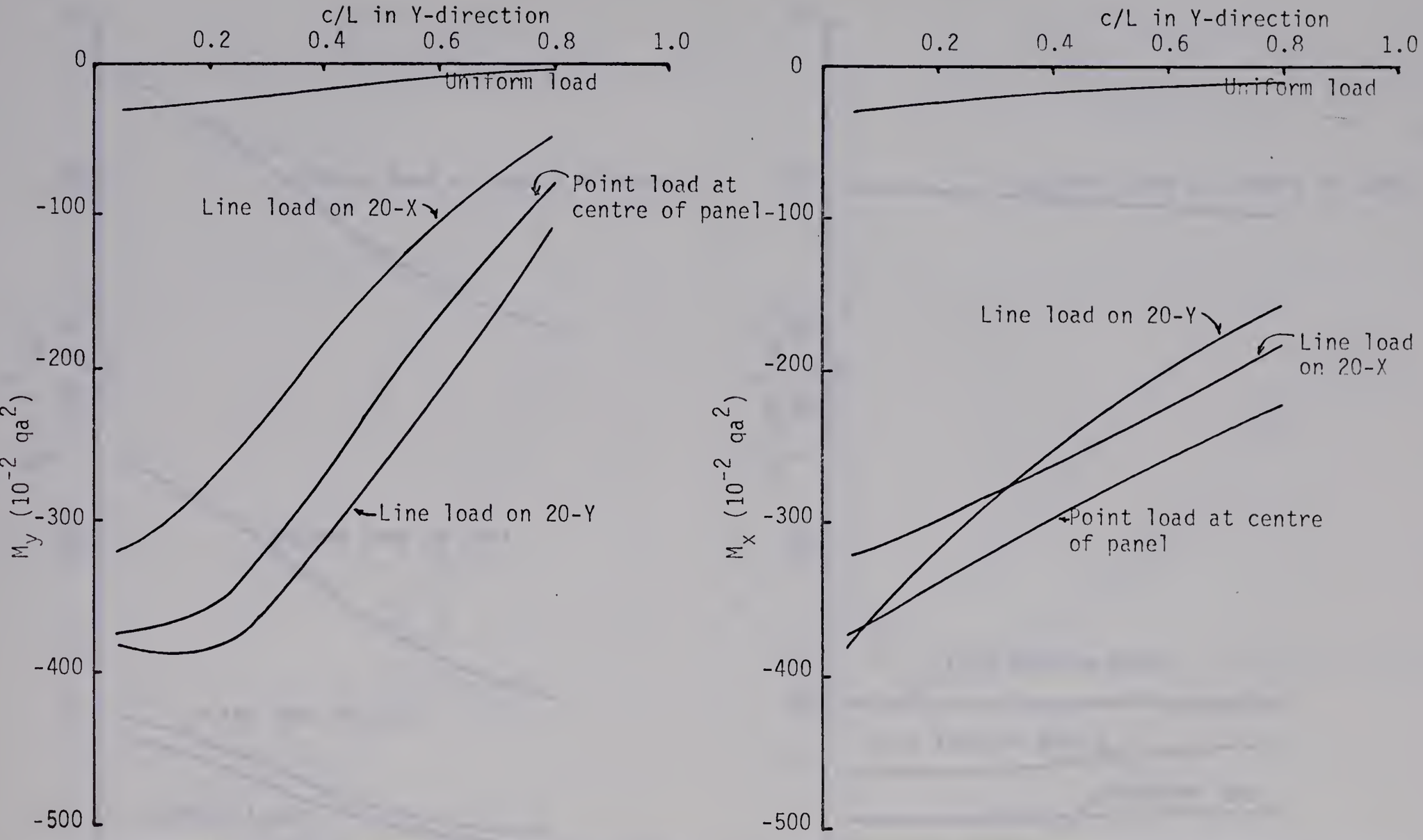
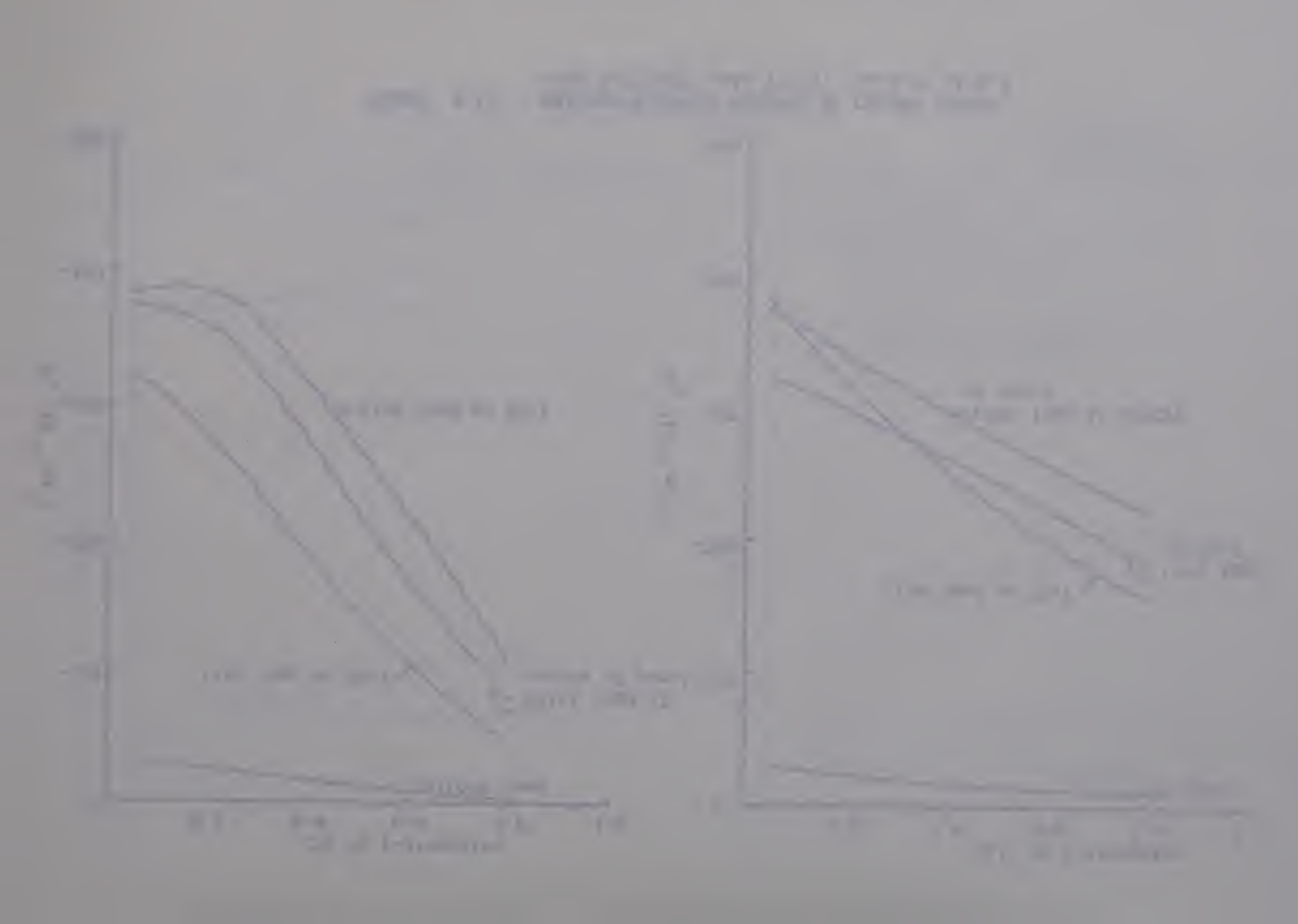
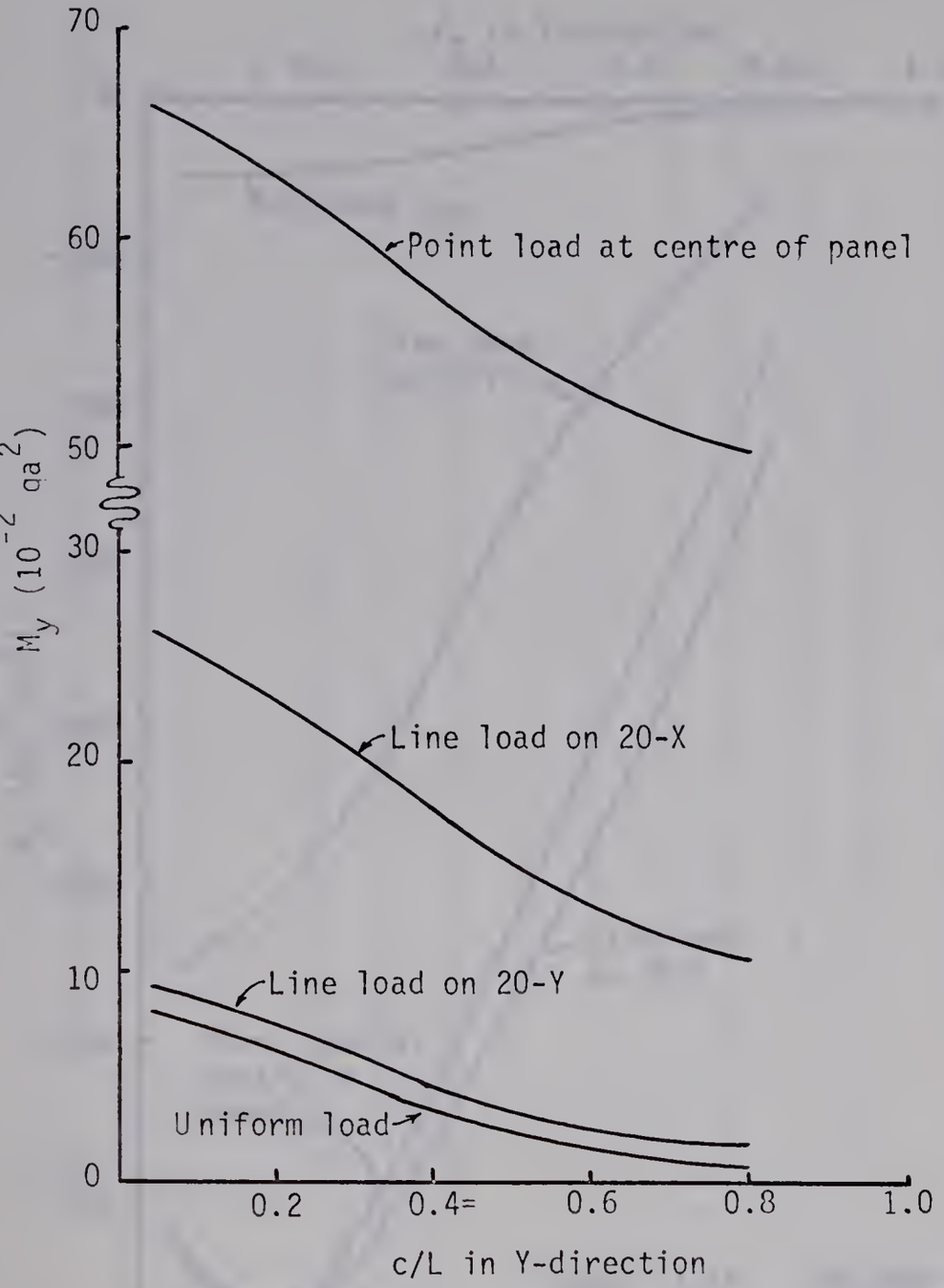


FIGURE 4.17 MAXIMUM NEGATIVE MOMENTS AT COLUMN CORNER
FOUR DIFFERENT LOAD TYPES; b/a=1.0; μ=0.2





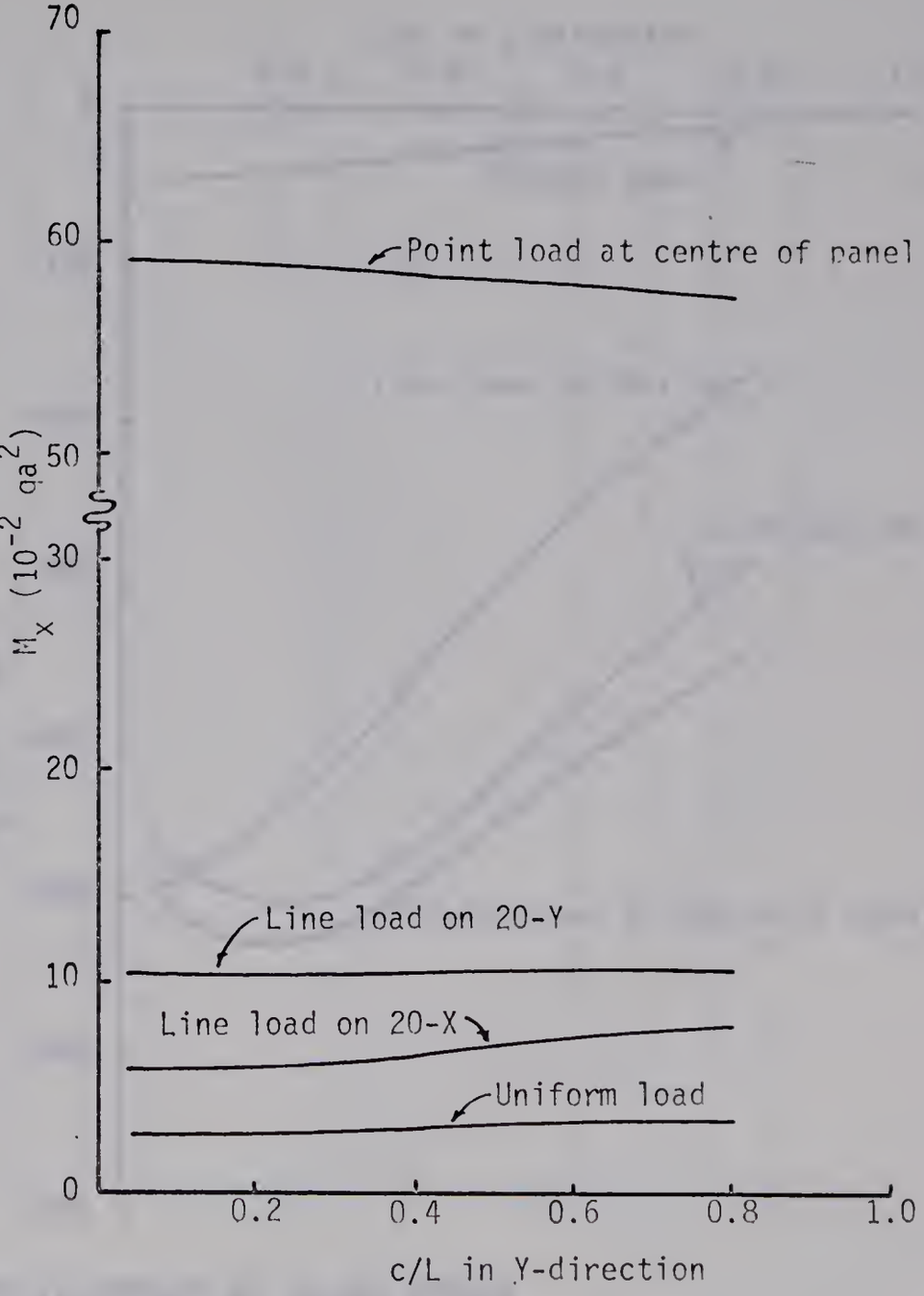
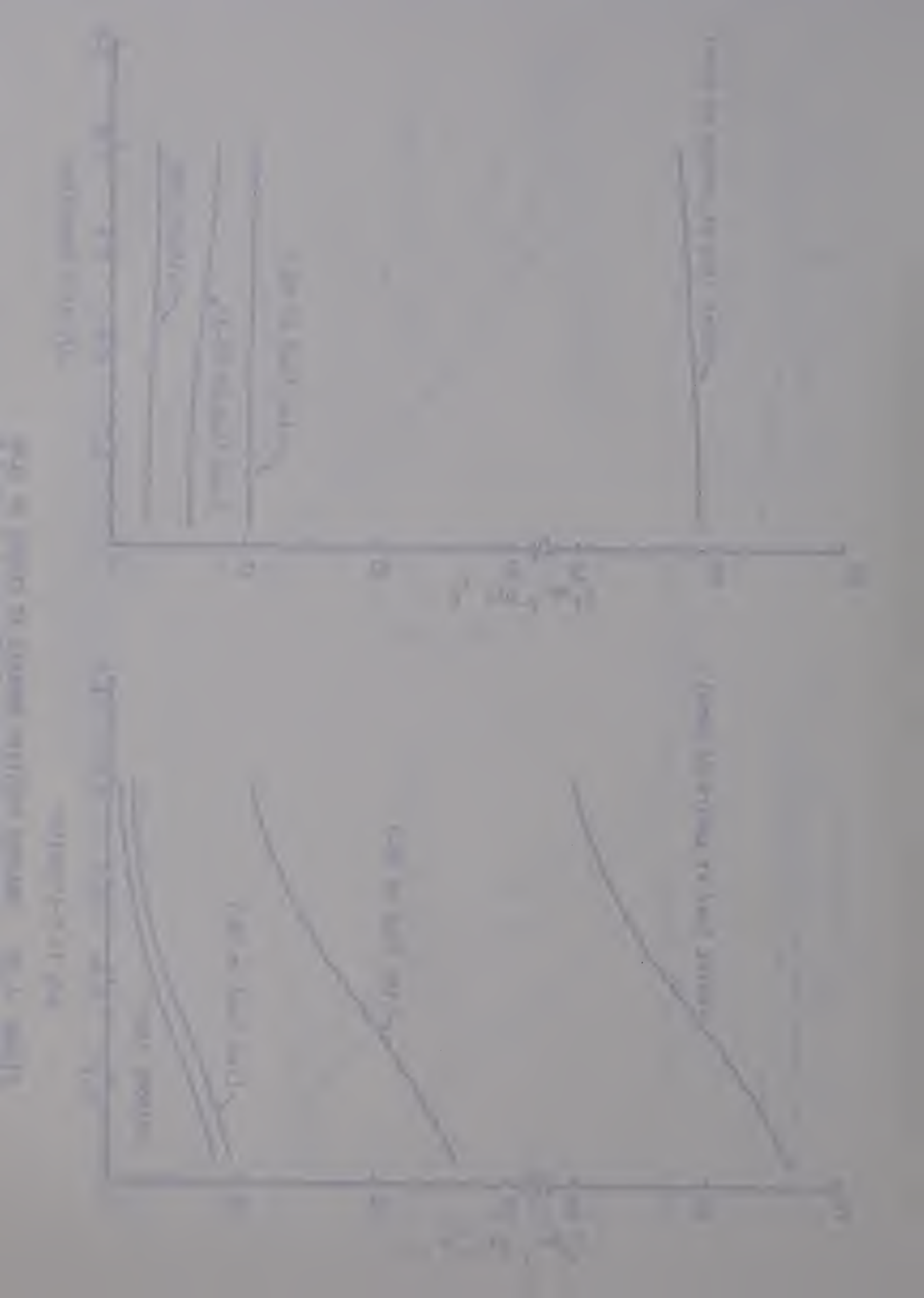
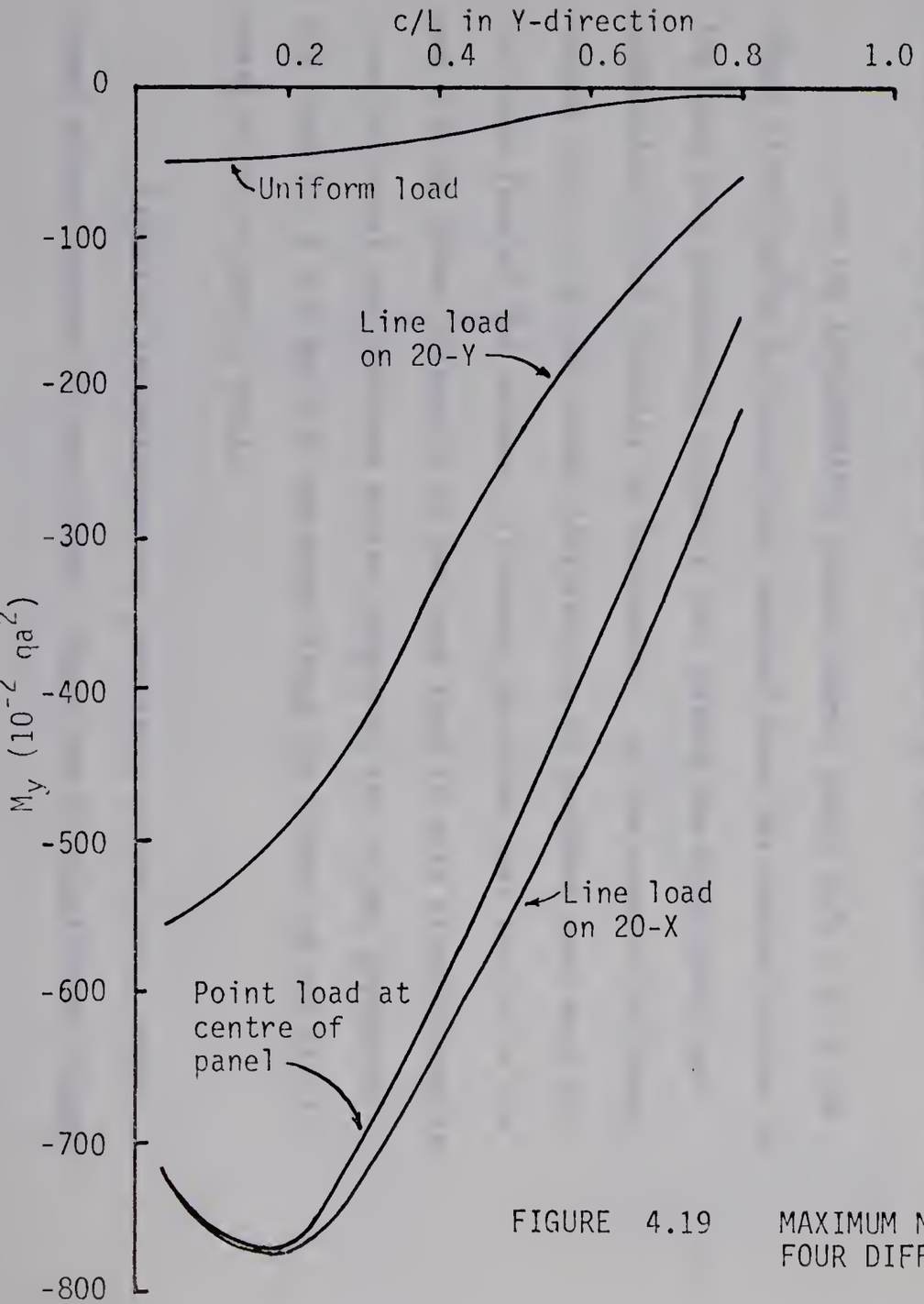


FIGURE 4.18 MAXIMUM POSITIVE MOMENTS AT CENTRE OF SLAB FOUR DIFFERENT LOAD TYPES; b/a=1.5; μ =0.2





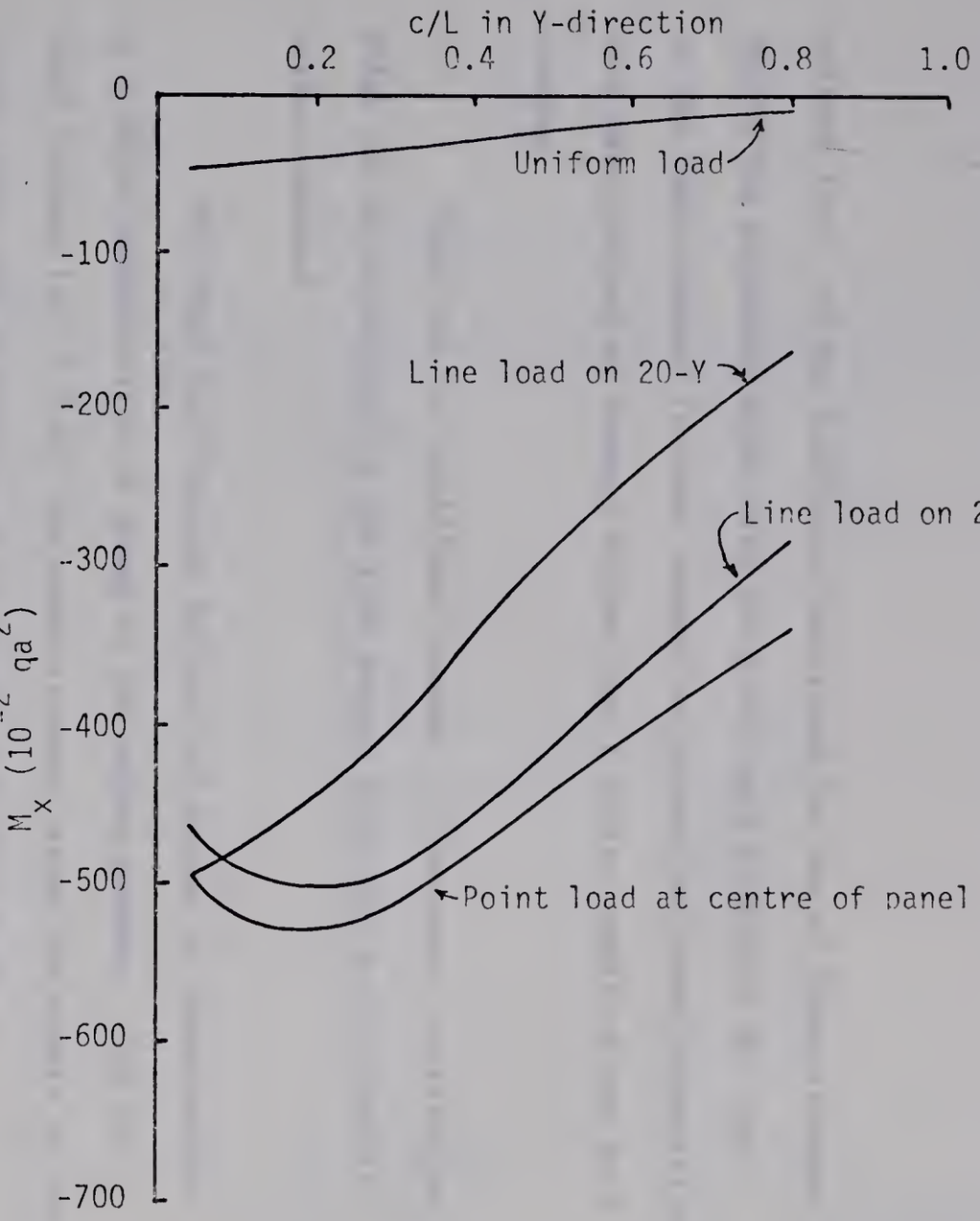
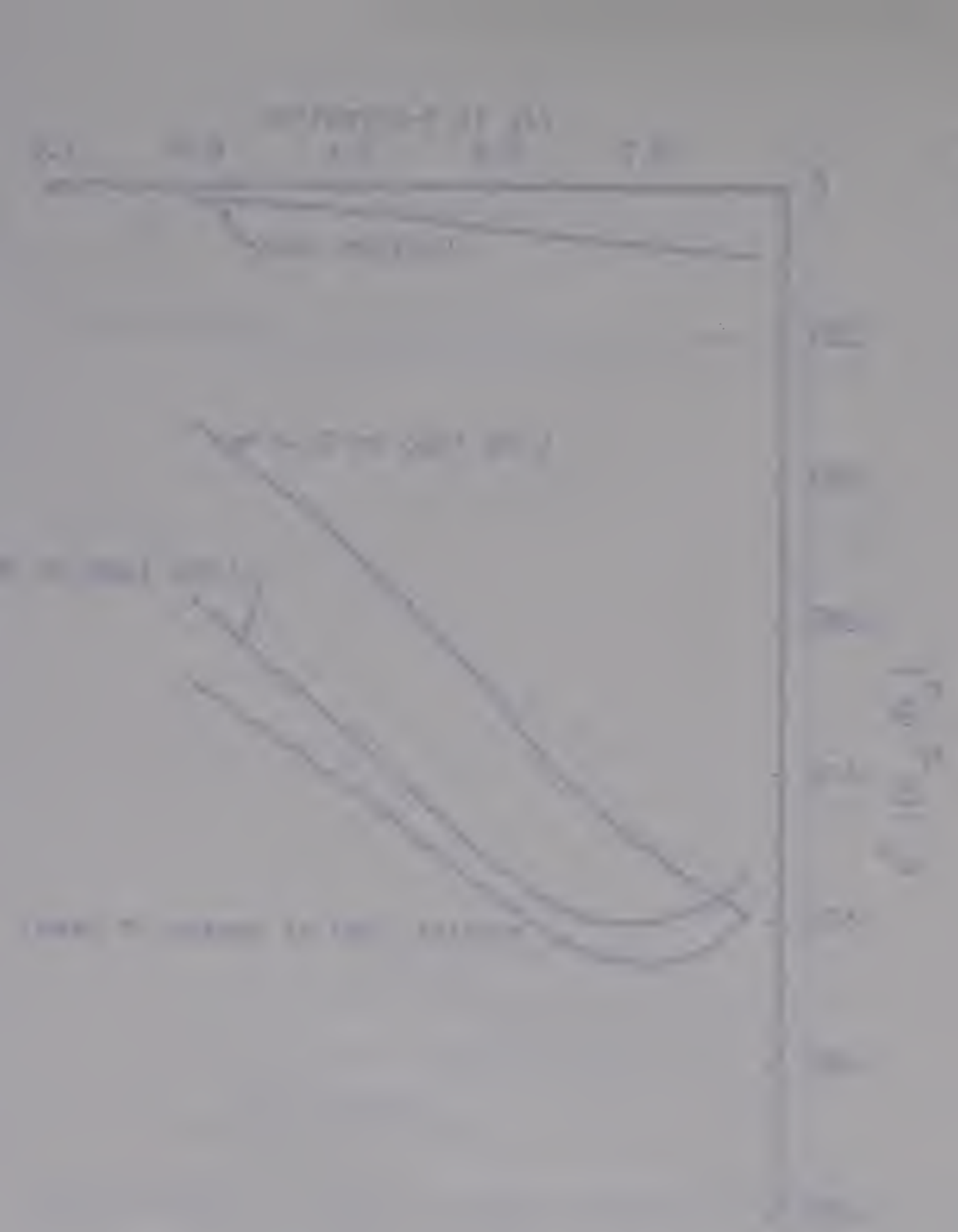
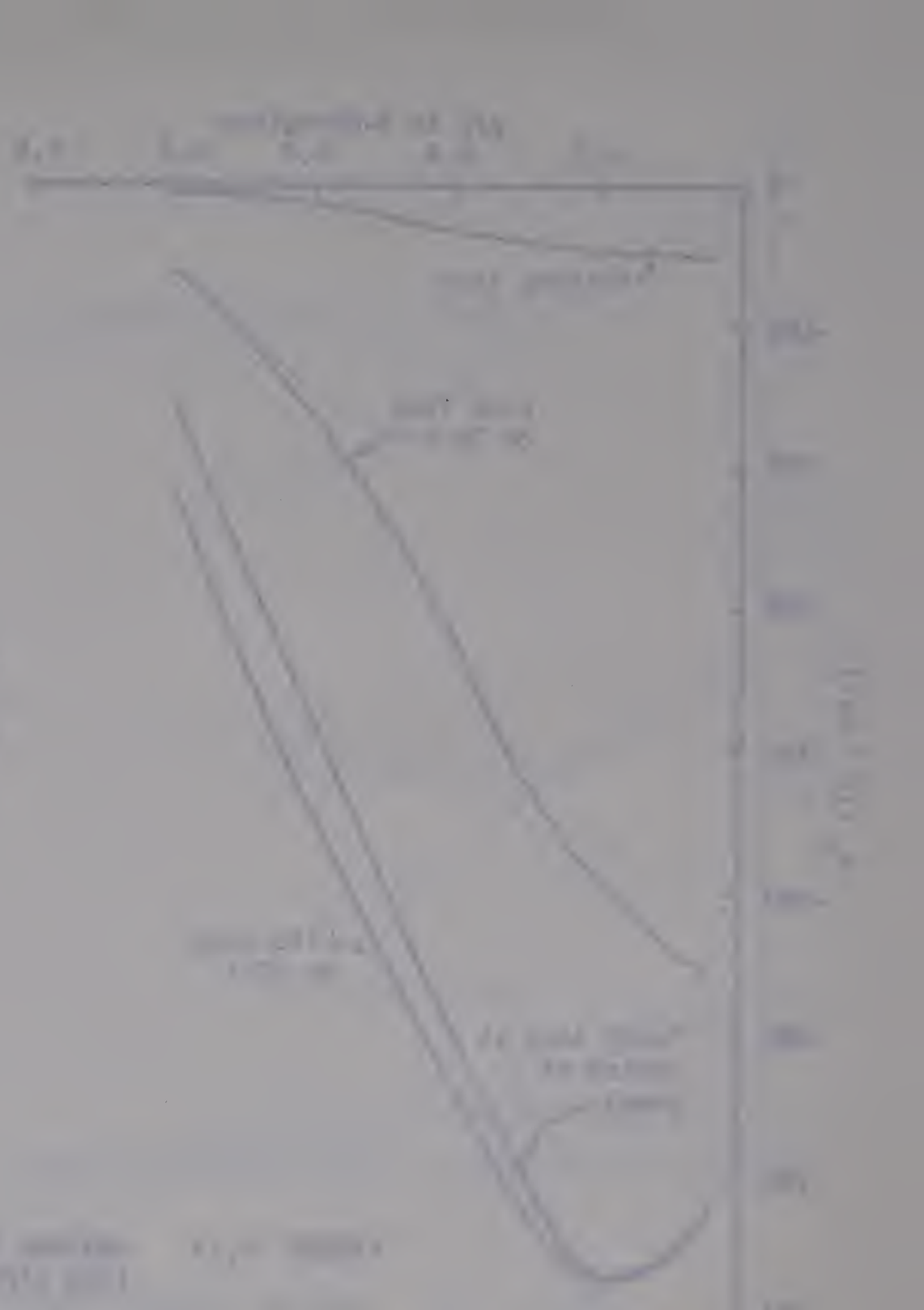


FIGURE 4.19 MAXIMUM NEGATIVE MOMENTS AT COLUMN CORNER
FOUR DIFFERENT LOAD TYPES; b/a=1.5; μ =0.2





uniform load. As was previously mentioned the sum of these stresses around the perimeter agree with the total load to within 3%. Due to the rapid changes in shear around the column the shear intensities have been plotted as average values over a finite length of one grid spacing.

There are no significant changes in the shear distribution which can be attributed to the slab aspect ratio for any of the b/a ratios examined.

The shear distribution for all c/L ratios is characterized by a large concentration of shear at the column corners. For the small columns (c/L = 0.05) the shear is small along the middle of the column faces and in some cases is negative in this region.

For the intermediate column aspect ratio (c/L = 0.4) the shear distribution is relatively constant over the centre portion of the long face decreasing slightly just before the high shear concentration in the vicinity of the corner. In the case of the long column (c/L = 0.8) the shear distribution is constant over most of the long face of the column. It should be noted that for c/L = 0.4 and 0.8 the shear intensity on the long face is only slightly smaller than the total load divided by the length of the column perimeter. For both c/L = 0.4 and 0.8 the shear along the middle of the short face of the column is small.

FIGURE 4.23 indicates the percentage of the total shear taken at the corners of the column. There are no significant changes

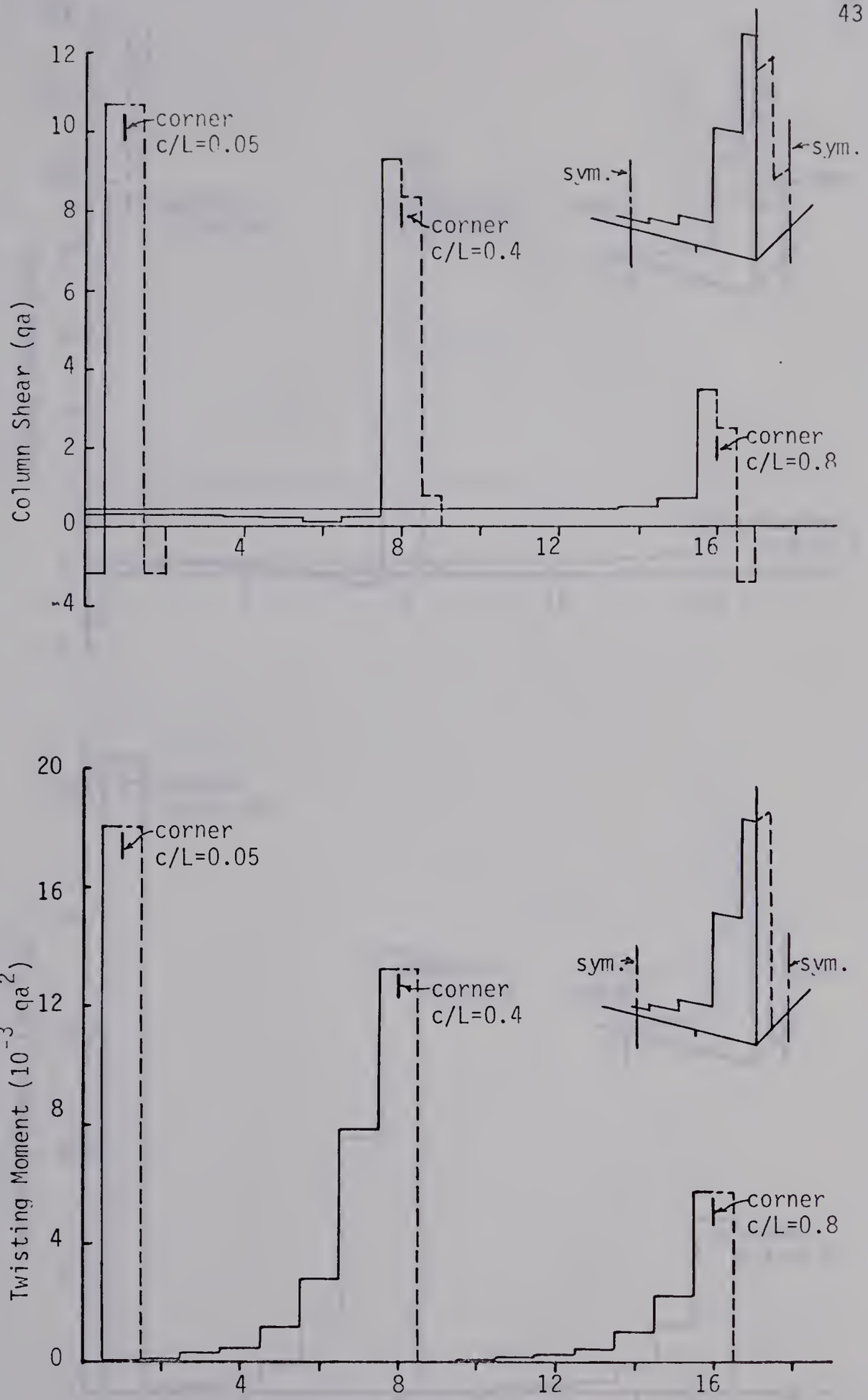
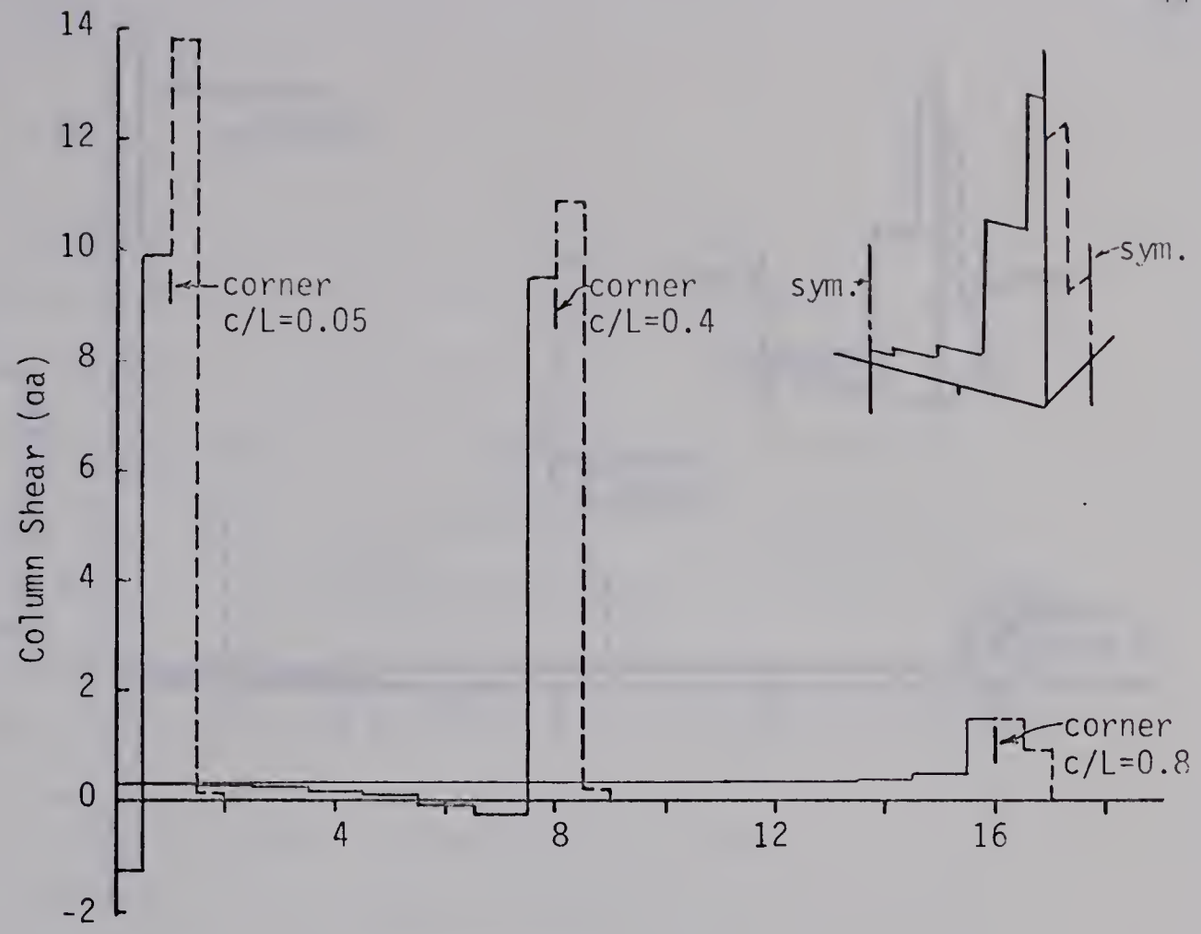


FIGURE 4.20 SHEARS AND TWISTING MOMENTS AROUND THE COLUMN
UNIFORM LOAD; b/a=1.0; μ=0.2







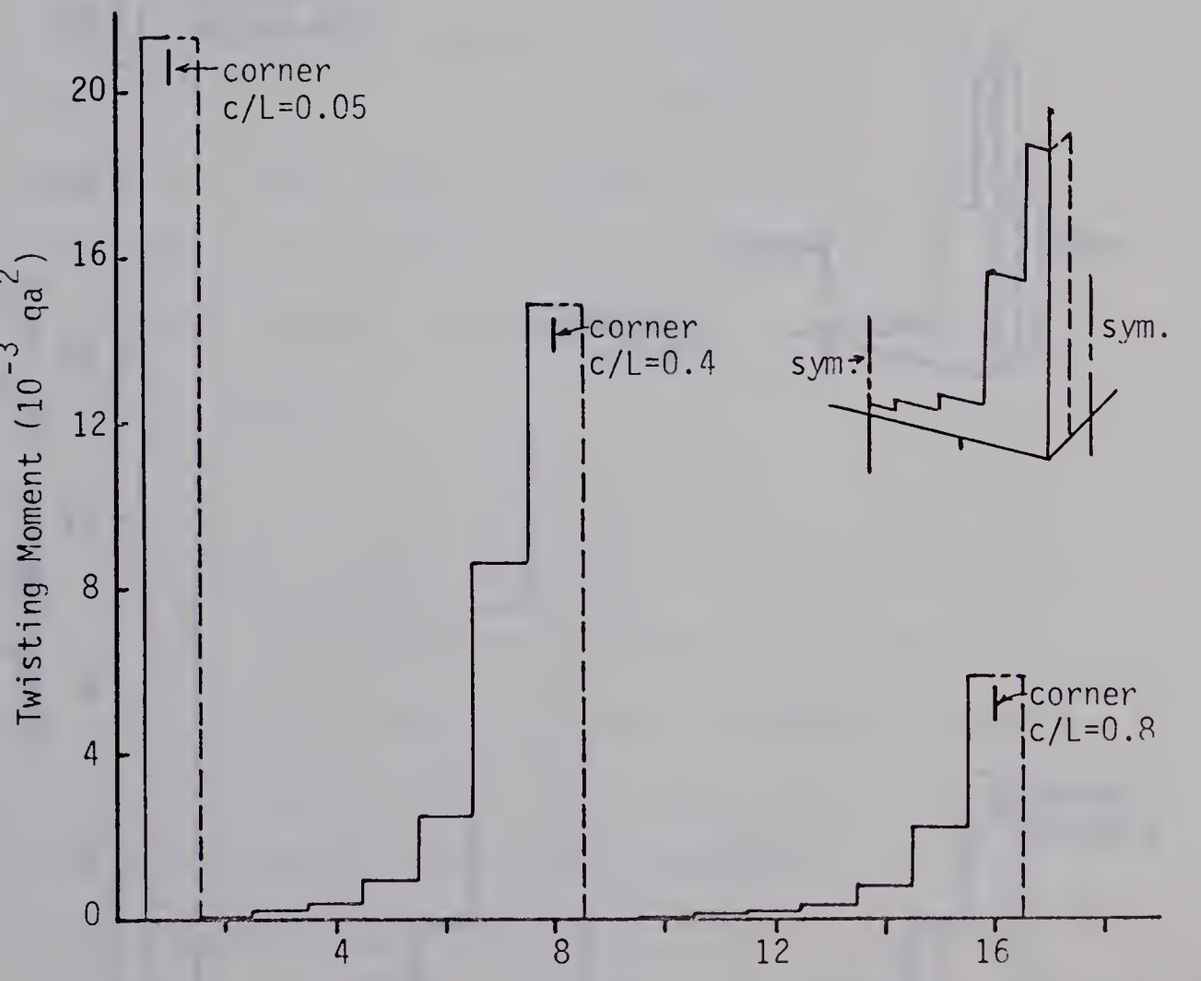


FIGURE 4.21 SHEARS AND TWISTING MOMENTS AROUND THE COLUMN
UNIFORM LOAD; b/a=1.2; μ=0.2





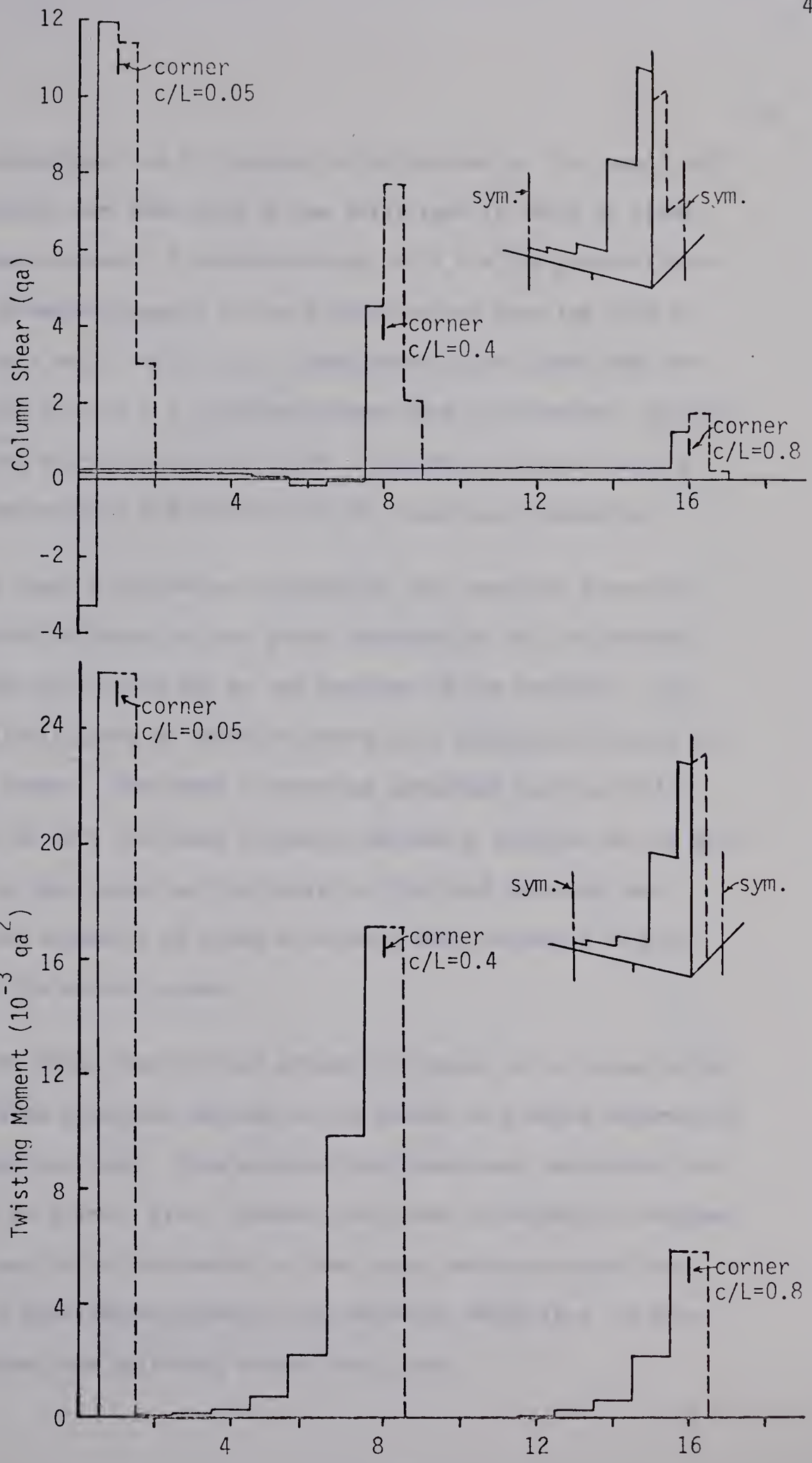


FIGURE 4.22 SHEARS AND TWISTING MOMENTS AROUND THE COLUMN
UNIFORM LOAD; b/a=1.5; μ=0.2



in these percentages due to changes in the b/a ratio. For small c/L ratios slightly more than 100% of the total load is taken as shear at the column corners. At approximately c/L = 0.4 the percent shear begins to decrease linearly to the minimum values when the slab is supported by a wall, (c/L = 1.0). The percent shear taken over the interval b/40 for c/L = 1.0 was calculated from equilibrium. The fact that it plots on the projection of the lines for c/L less than 1.0 further substantiates the accuracy of the numerical procedures.

A shear distribution obtained by this analysis gives only a qualitative indication of the shear distribution in a reinforced concrete flat plate slab due to two features of the analysis. Any elastic analysis gives an infinite stress at a discontinuity such as the column corner. Thus when a numerical procedure such as finite differences is used the shear intensity becomes a function of the grid spacing. In this case, had the length of the grid division been increased the intensity of shear would have been decreased in the vicinity of the column corner.

The shear distribution around the column in an actual reinforced concrete structure depends on the amount of plastic deformation in the column and slab. This analysis only considers non-deflecting columns and no plastic flow. However the shear distributions obtained do give a qualitative indication of the shear behavior around the column until substantial plastic flow occurs at which time the shear is distributed more uniformly around the column.

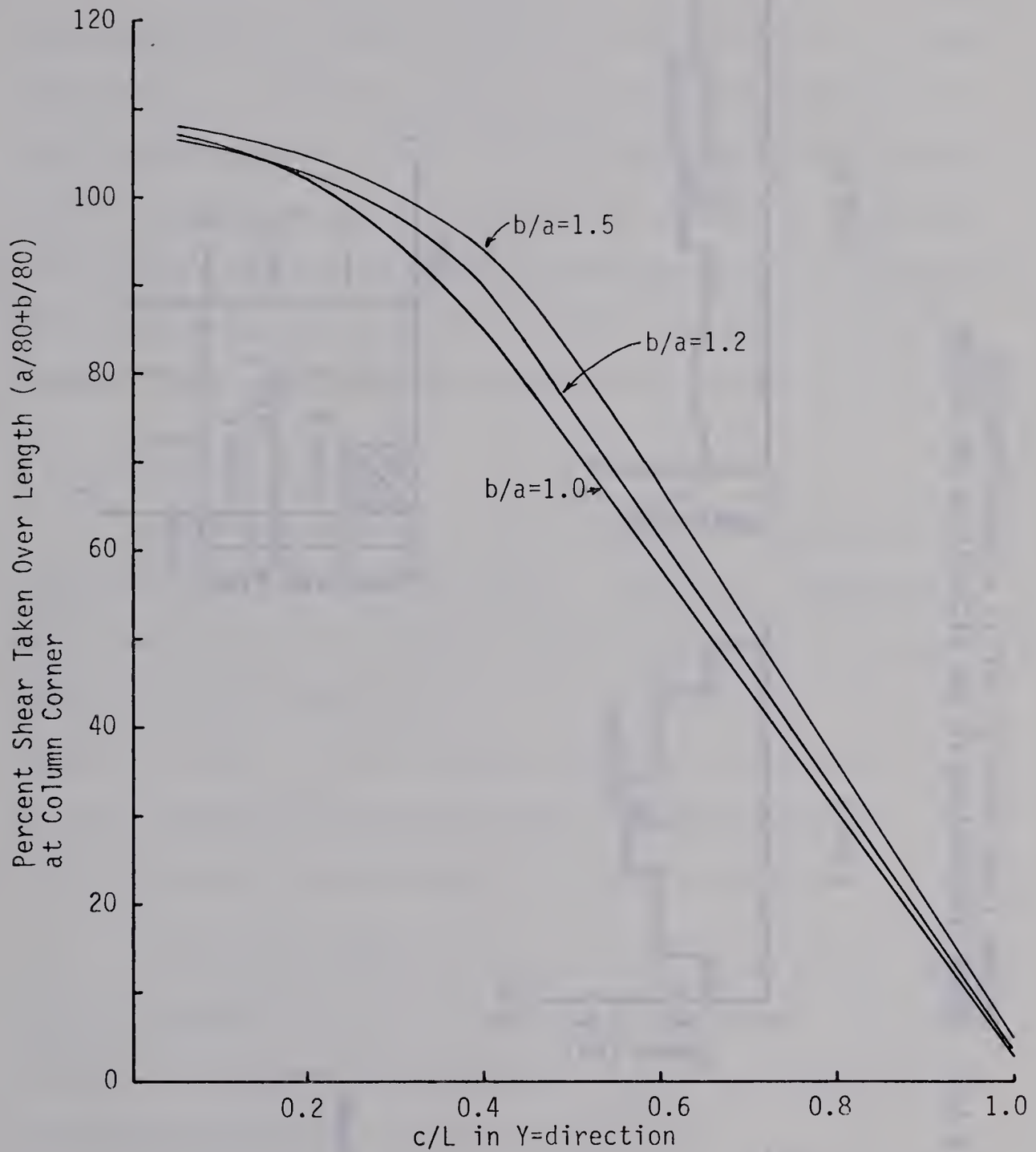
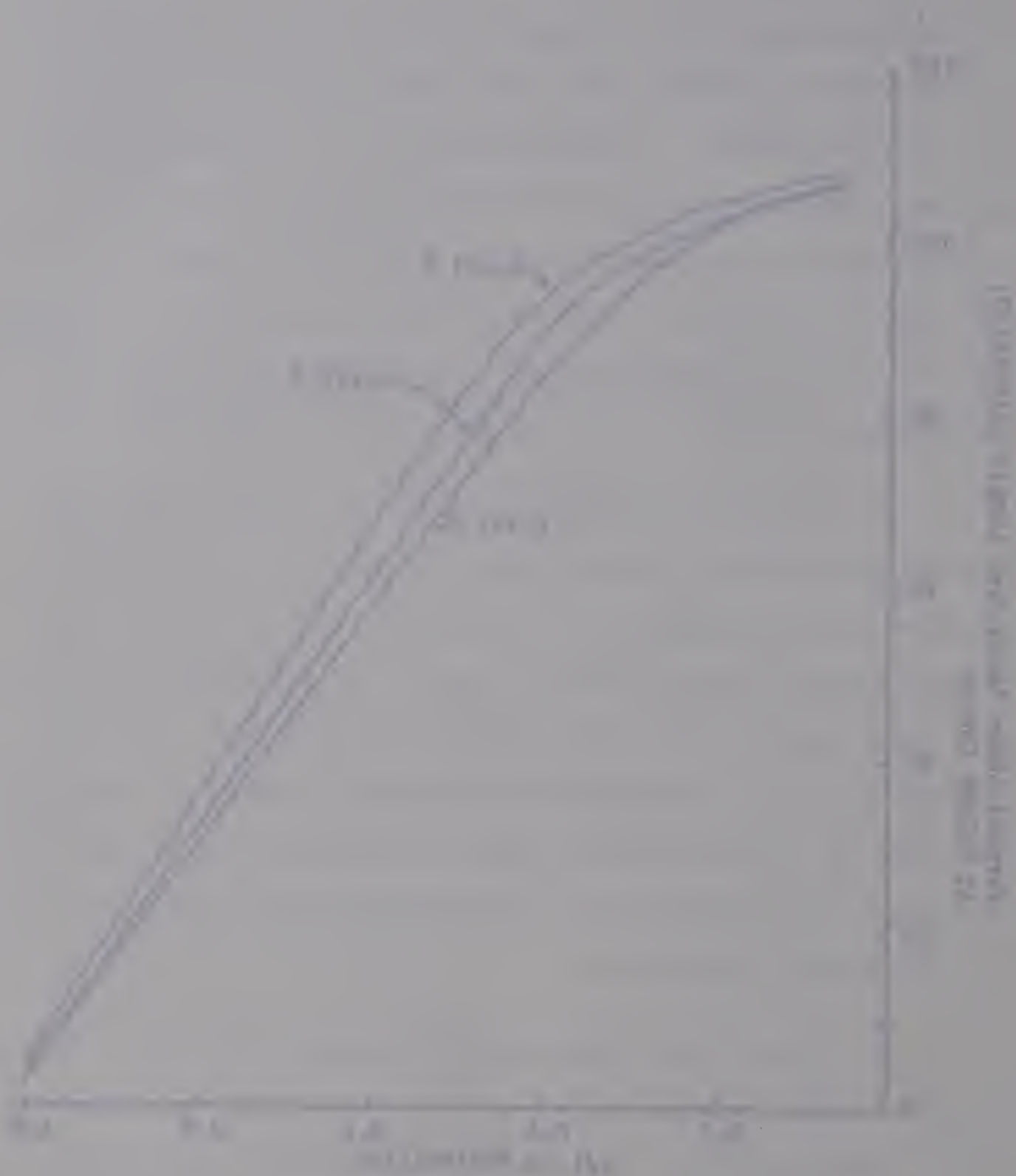


FIGURE 4.23 PERCENT SHEAR TAKEN AT COLUMN CORNER
UNIFORM LOAD; b/a=1.0,1.2,1.5



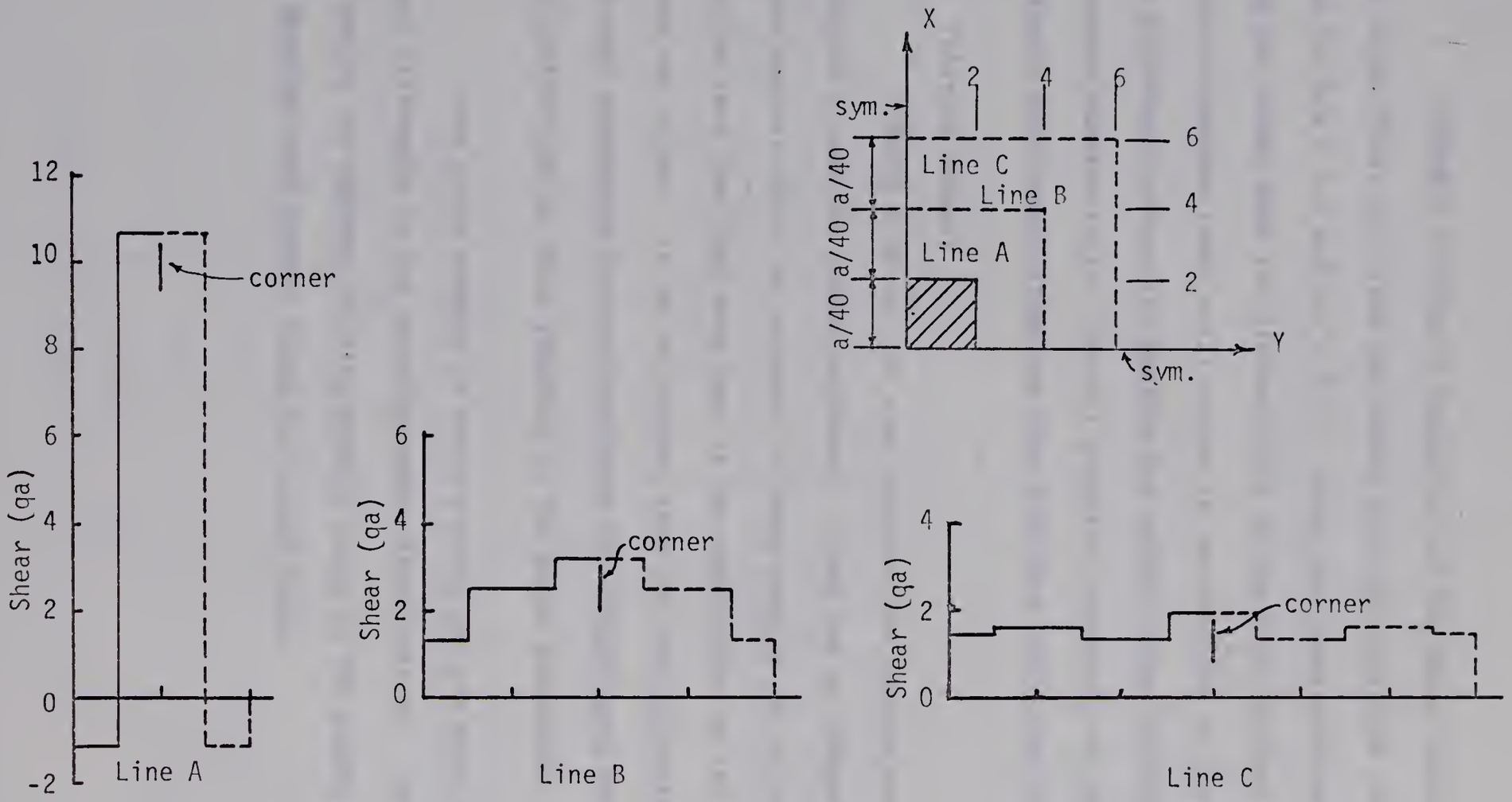


FIGURE 4.24 SHEAR DISTRIBUTION IN THE VICINITY OF THE COLUMN
UNIFORM LOAD; b/a=1.0; c/L in Y-direction=0.05



FIGURE 4.24 shows a comparison of the shear distribution at the column face, a/40 from the column face and a/20 from the column face for b/a = 1.0 and c/L = 0.05. Along the lines a/40 and a/20 from the column face the discontinuity in the slab has been removed. While the maximum shear still occurs in the vicinity of the corner, the difference between the maximum and minimum shear intensities has decreased substantially. For all practical purposes, the shear is uniformly distributed along the line a/20 from the column face.

4.5 Twisting Moments

FIGURES 4.20 to 4.22 also indicate the twising moment distribution around the column perimeter. There are no independent values against which the accuracy of these results can be checked other than the fact that they were used in the calculation of the shears around the column. It can be assumed that for the application to an actual reinforced concrete structure the results have the same limitations as those reported in the shear discussion.

Once again changes in the b/a ratio did not make any significant difference in the twisting moment distributions. For each of c/L ratios the maximum twisting moments occur at the column corners and decrease very rapidly along the column faces.

CHAPTER V

CONSIDERATIONS FOR DESIGN

5.1 Introduction

The present ACI Code makes no reference to the use of rectangular columns in the design of flat plate slabs. This no doubt stems from the fact that the use of rectangular columns is a relatively recent architectural innovation. The designer's main concern is whether the Code definitions give a safe approximation of the total design moment (M_o). Also the question arises as to how the total moment should be divided, in the sense of strips perpendicular to the long face of the column, since obviously the writers of the present code did not envision this condition.

All discussion has been limited to the case of rectangular slabs with uniformly distributed loads and with a Poisson's ratio of 0.2. Results have been presented for both b/a = 1.0 and 1.5. FIGURE 5.1 illustrates the system used to present the moments across critical lines in the slab.

5.2 Total Design Moment

The ACI Building Code (318-63) defines the total design moment in a panel as M_o. From the equation given in clause 2101(e)

$$M_o = 0.10WLF\left(1 - \frac{2c}{3L}\right)^2$$

where $F = \left(1.15 - \frac{c}{L}\right)$ but not less than 1.0

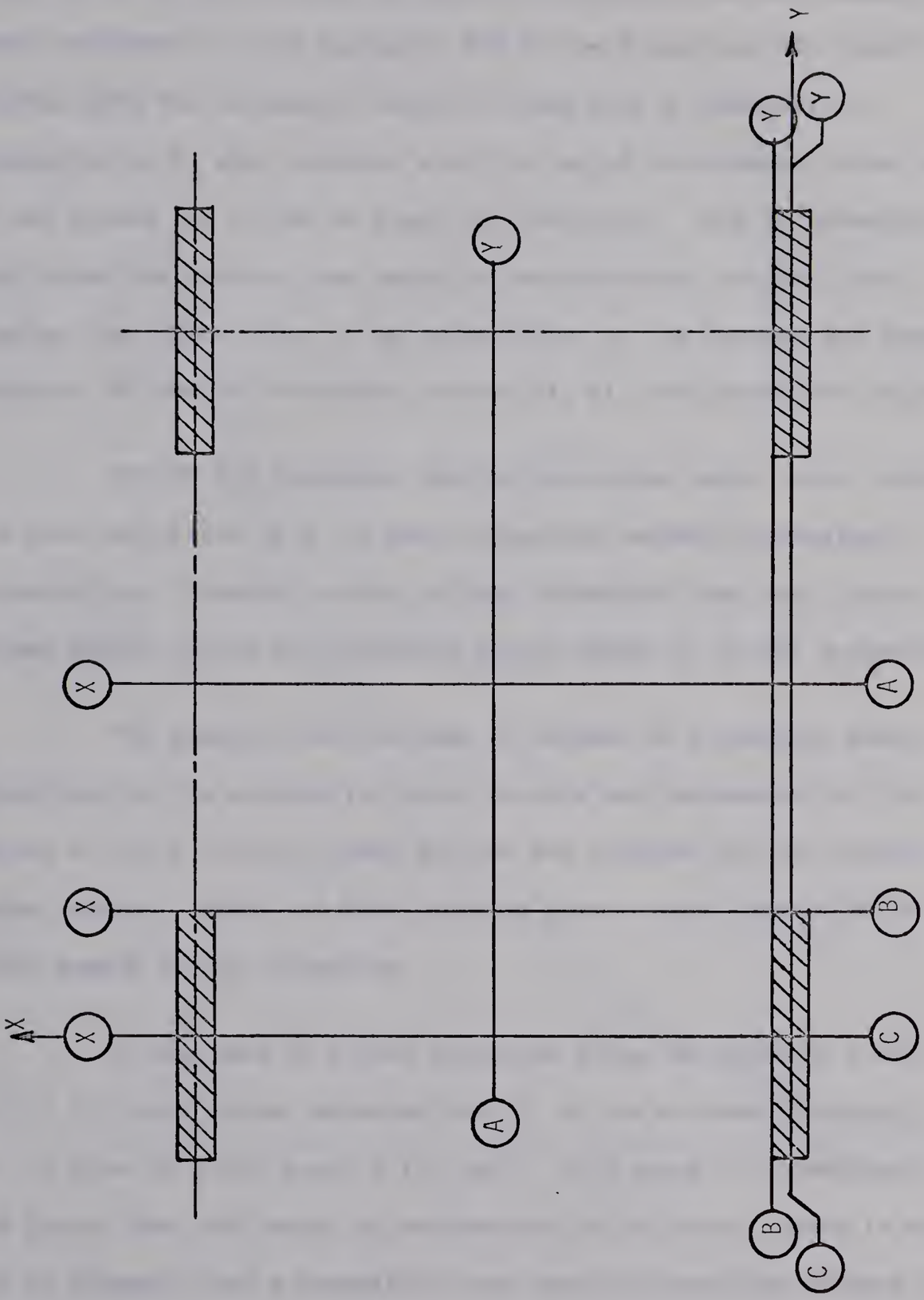
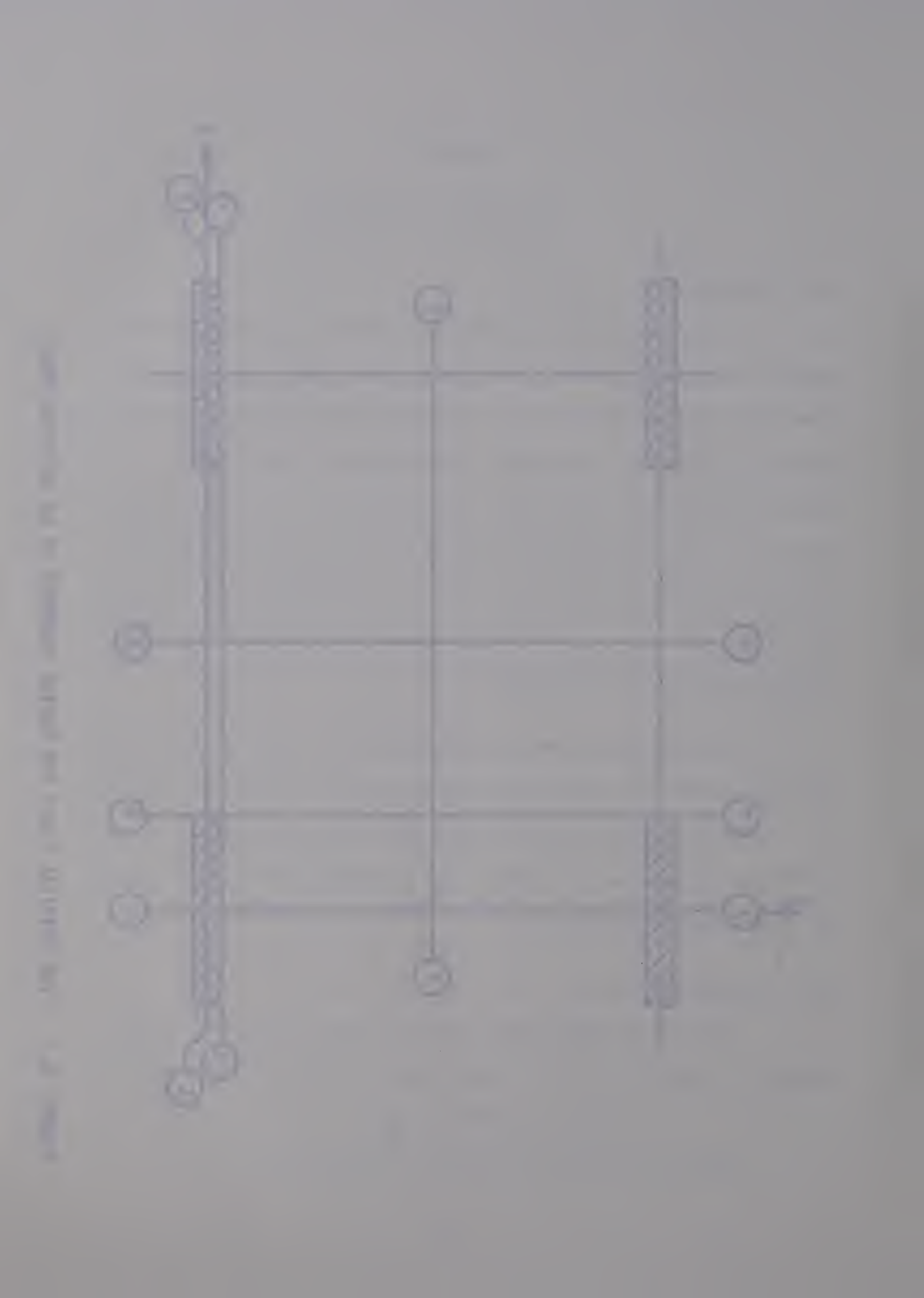


FIGURE 5.1 THE CRITICAL LINES FOR DESIGN PURPOSES IN AN INTERIOR PANEL



This equation has no rigid mathematical derivation and has commonly been considered to only represent 80% of the total statical moment. However with the included F factor it does give a conservative indication of M_o when compared with the sum of the moments across AY, CY and around the column as found by this study. This is understandable since the analysis was based on non-deflecting columns, thus causing the column shear to be concentrated at the corners and thereby reducing the sum of the moments across AY, CY, and around the column.

FIGURE 5.2 indicates that as the column aspect ratio increases the code definition of M_o in the y-direction becomes increasingly conservative. However it must be also remembered that with larger column aspect ratios the principal design moment is in the x-direction.

The analysis was confined to columns of a constant width. Therefore, in the x-direction total moments are represented by the values at c/L = 0.05 on FIGURE 5.2 and are constant for all column aspect ratios. Again the code equation gives a good prediction of the total moment in this direction.

In the case of a slab supported along two sides by a wall (c/L = 1.0) equilibrum indicates that M_o in the x-direction should be, in terms of total span, $0.113\ qab^2$. This value is approximately 10% higher than the design M_o recommended by the Code. There is room for an argument that a transition zone should be provided between the predominantly two-way slab action at low c/L ratios and the one-way slab action at high c/L ratios to ensure uniformity in the factor of safety. However in the y-direction the Code is conservative and a

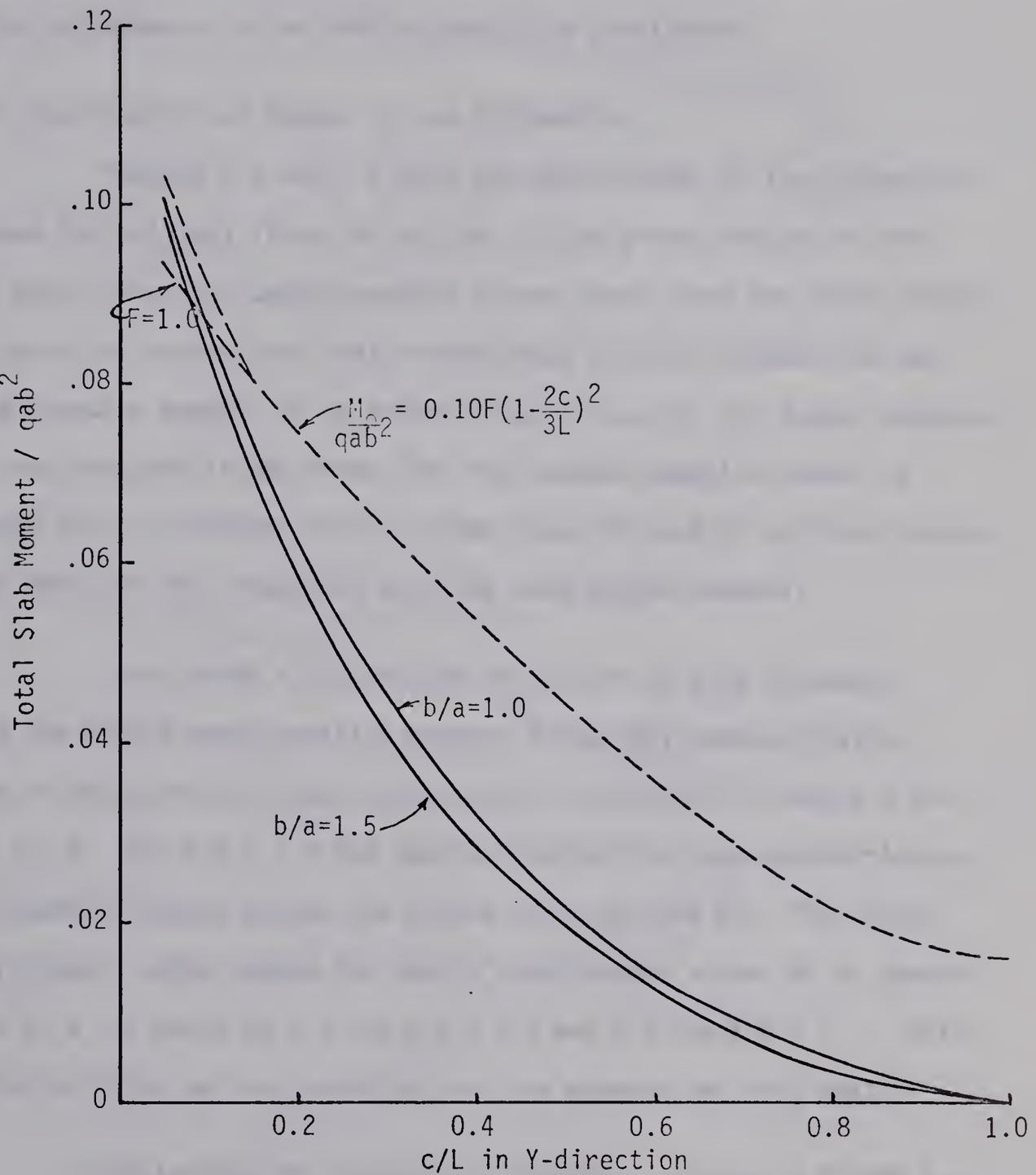
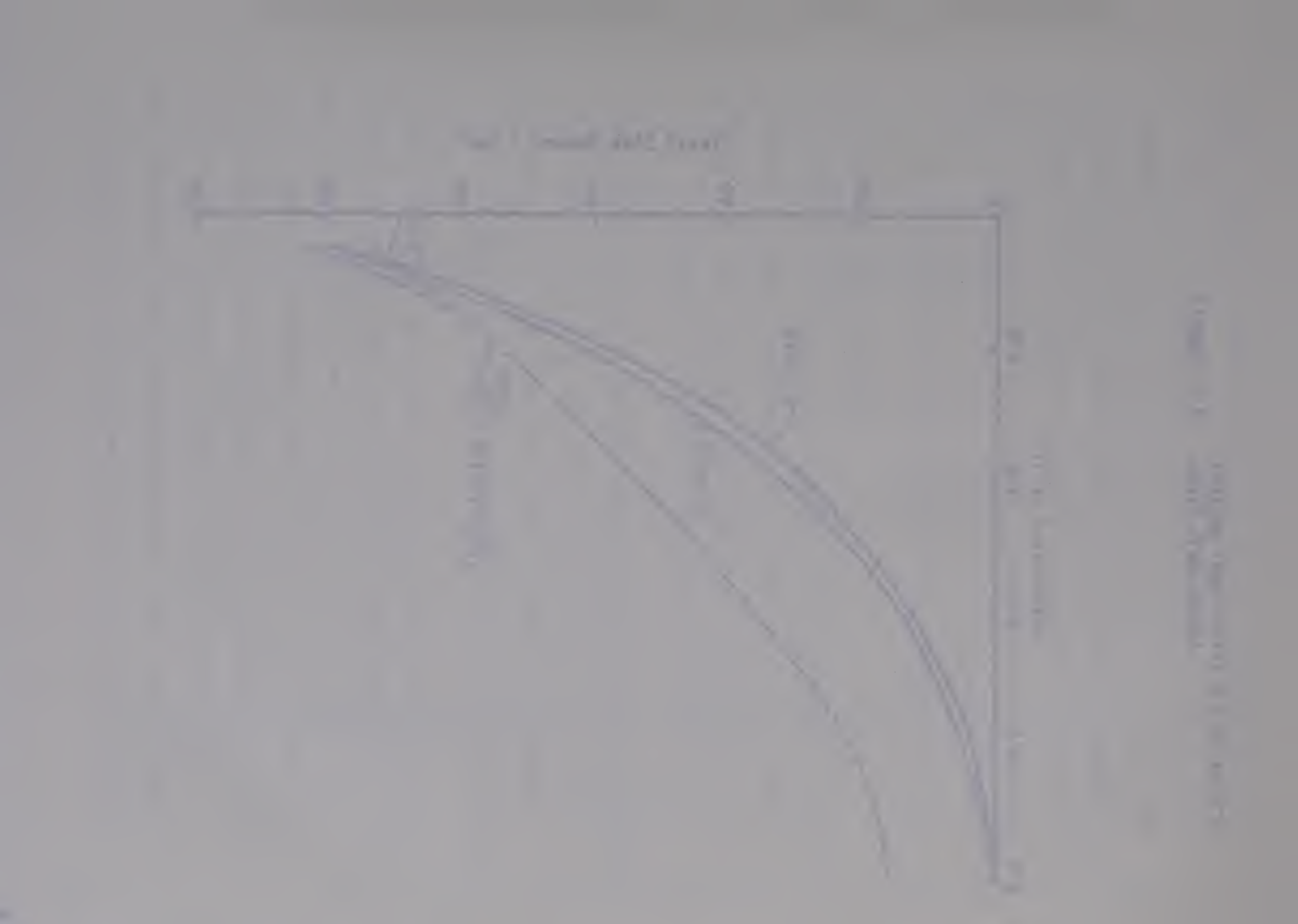


FIGURE 5.2 TOTAL SLAB MOMENT
UNIFORM LOAD; b/a=1.0,1.5; μ=0.2



redistribution of moment to the y-direction is possible for intermediate column aspect ratios. At high values of the column aspect ratio an increase in the Code M_o should be considered.

5.3 Distribution of Moment in the Y-direction

FIGURES 5.3 and 5.4 show the total moment in the y-direction across the critical lines, AX and BX, in the strips defined by the ACI Code. The code design moments across these lines are shown dotted. It should be pointed out that conventional practice assumes the maximum negative moments to be across column line, CX, for design purposes. In this analysis it was found that the maximum negative moment is across the line defined by the column face, BX, and it is these moments that are used for comparison with the code design moments.

From FIGURE 5.3, the code values are in good agreement with the middle strip positive moments across AX, underestimating them slightly for c/L ratios less than 0.3 for b/a = 1.0 and 0.4 for b/a = 1.5. For b/a = 1.5 and small c/L ratios the Code underestimates the negative moment across the middle strip of line BX. The effect of Poisson's ratio causes the middle strip moment across BX to change sign at a c/L ratio of 0.4 for b/a = 1.0 and 0.5 for b/a = 1.5. This is not critical as the resulting positive moments are very small.

Considering the column strip moments as shown in FIGURE 5.4, the positive moments from the Code are in good agreement with the analytical values up to a c/L ratio of 0.4 above which the Code is conservative. With respect to the negative column strip moments

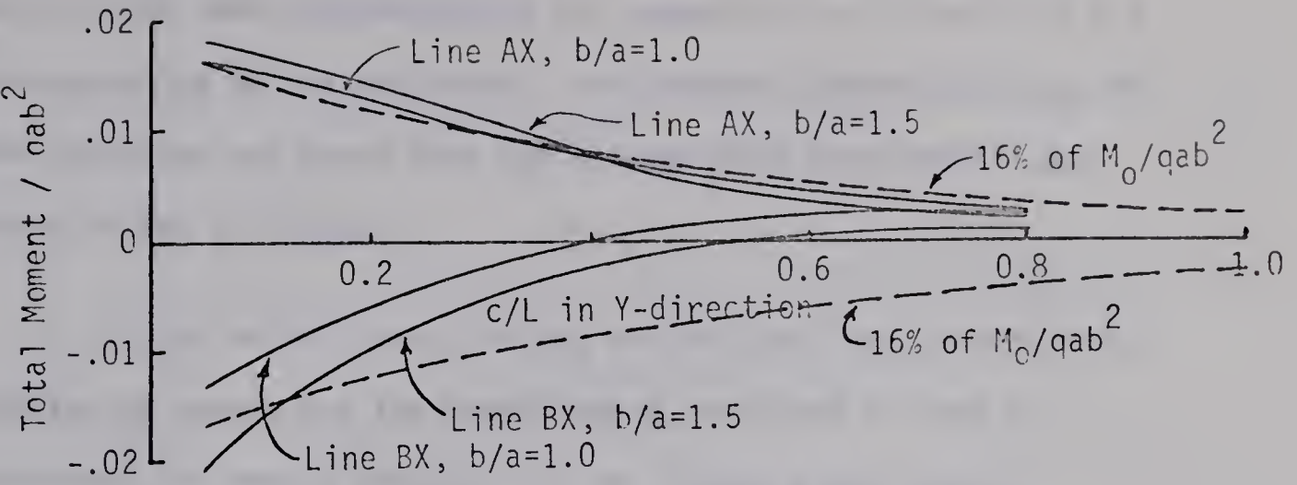


FIGURE 5.3 MIDDLE STRIP MOMENTS IN Y-DIRECTION
STRIP WIDTH=0.5a

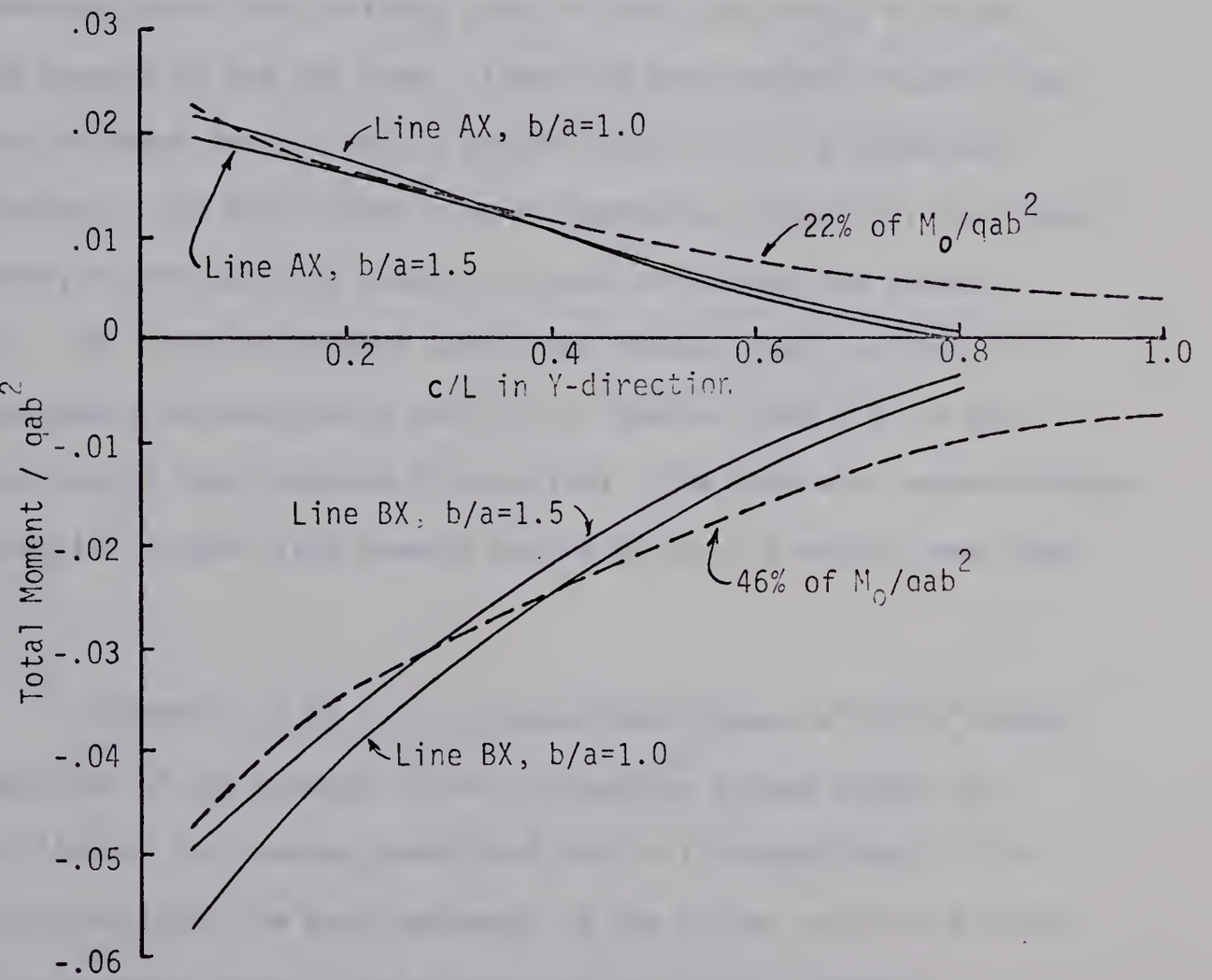
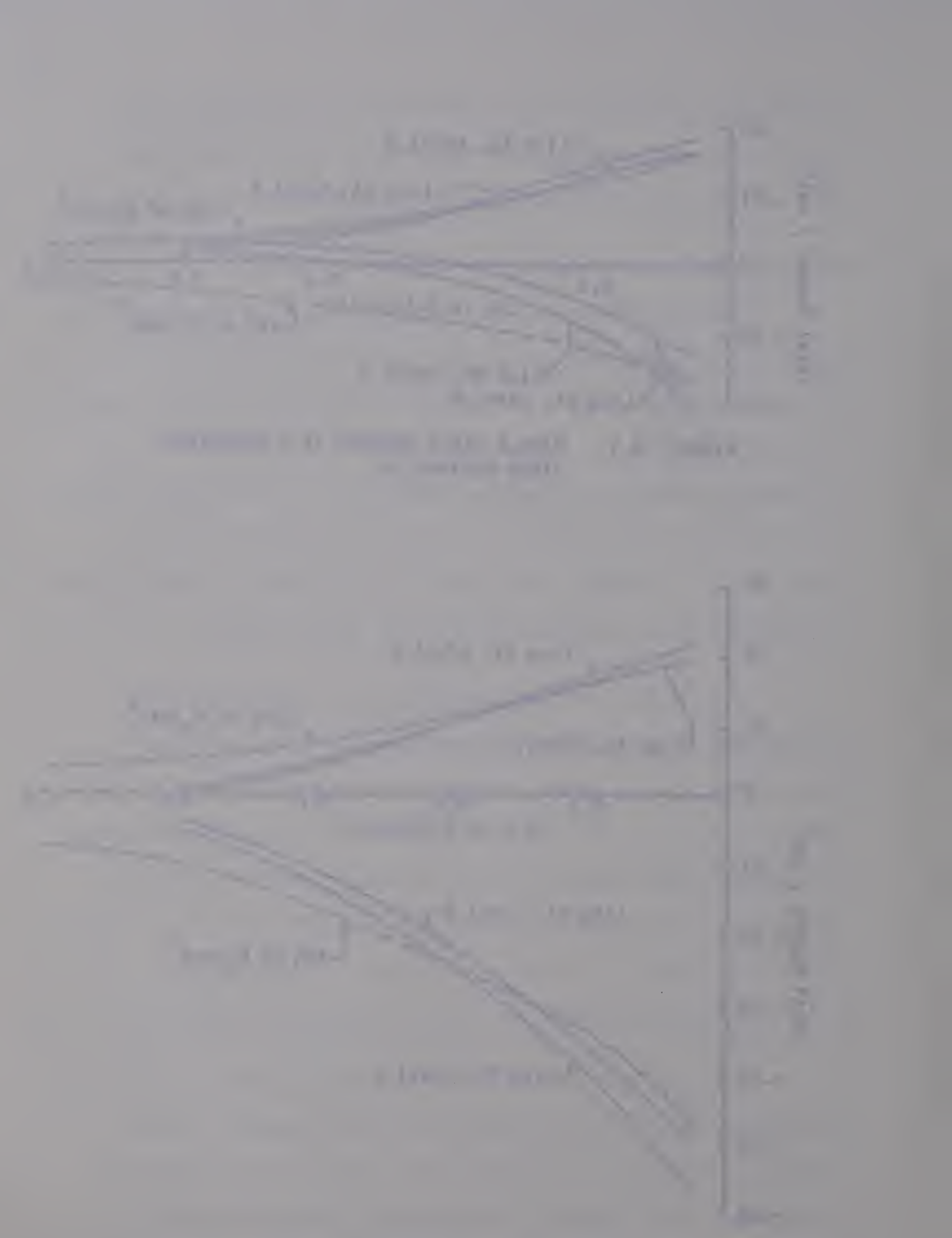


FIGURE 5.4 COLUMN STRIP MOMENTS IN Y-DIRECTION
STRIP WIDTH=0.5a



across BX, the Code underestimates the moments to a c/L ratio of 0.3 to 0.4 depending on the b/a ratio: the maximum difference being 17%. However practice has shown that the present code requirements are adequate at low c/L ratios.

It may be said that, in the y-direction, the present code definition of strips and the portion of M_o assigned to them are satisfactory for design purposes for any column aspect ratio.

5.4 Distribution of Moment in the X-direction

FIGURES 5.5 and 5.6 compare the total moments in the x-direction across the critical lines in the slab strips with the design moments of the ACI Code. Again the code definition of strips is used although for c/L ratios greater than 0.5 it is physically meaningless. The Code values give a reasonable indication, for design purposes, of the positive moments in both the middle and column strips. The negative moments across the middle strip portion of BY are generally underestimated and for c/L greater than 0.5 the Code underestimates these moments by over 100%. The Code also underestimates the negative column strip moments across BY for c/L ratios less than 0.4.

FIGURES 4.12 to 4.14 indicated that there is little change in magnitude of the moments in the x-direction across either the centre line or the quarter line (20-Y and 10-Y respectively). This partially explains the good agreement of the column and middle strip positive moments across line AY with the Code design moments.

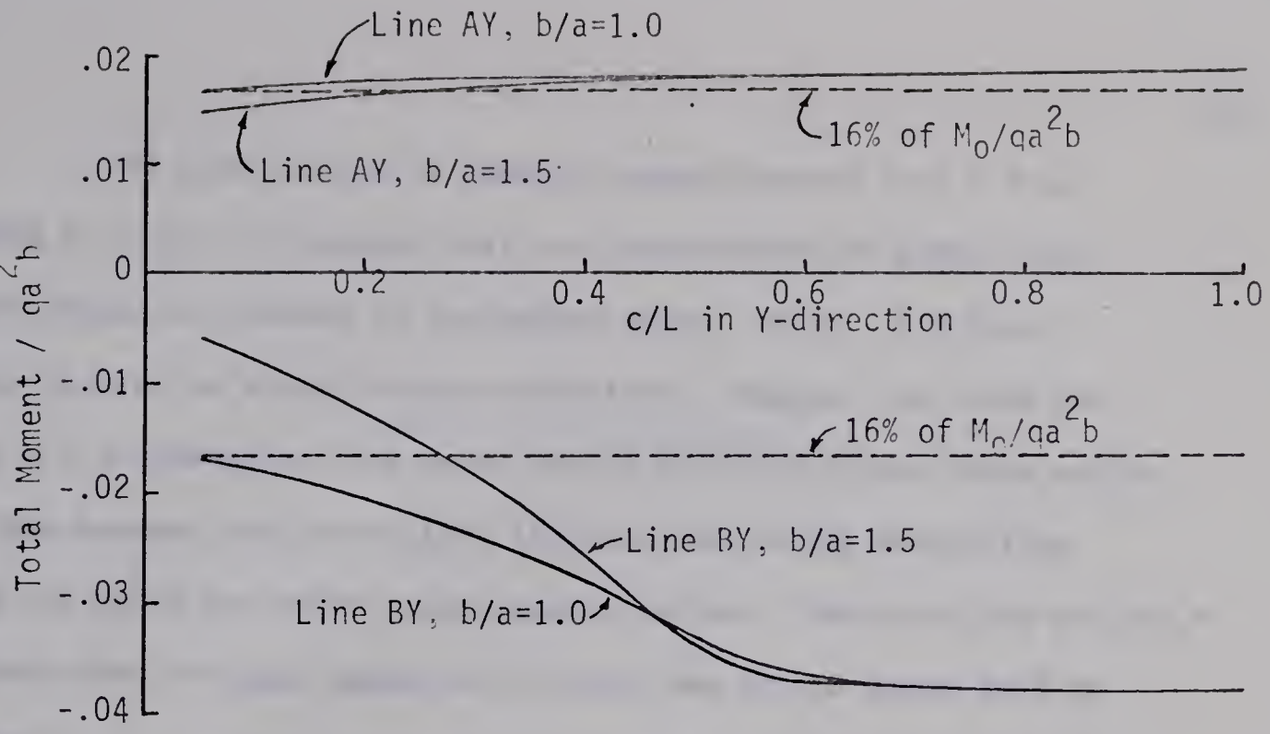


FIGURE 5.5 MIDDLE STRIP MOMENTS IN X-DIRECTION
STRIP WIDTH=0.5b

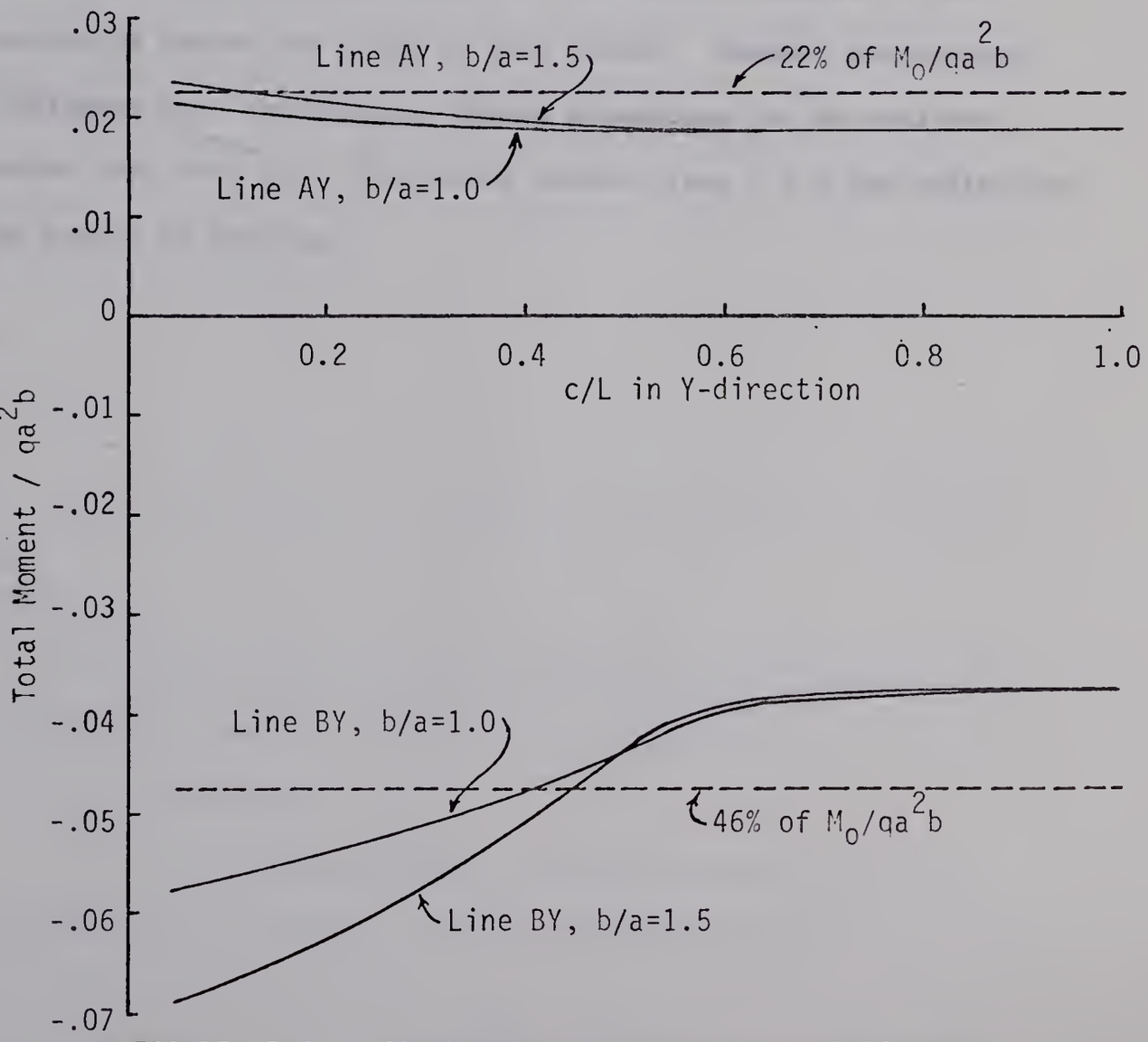
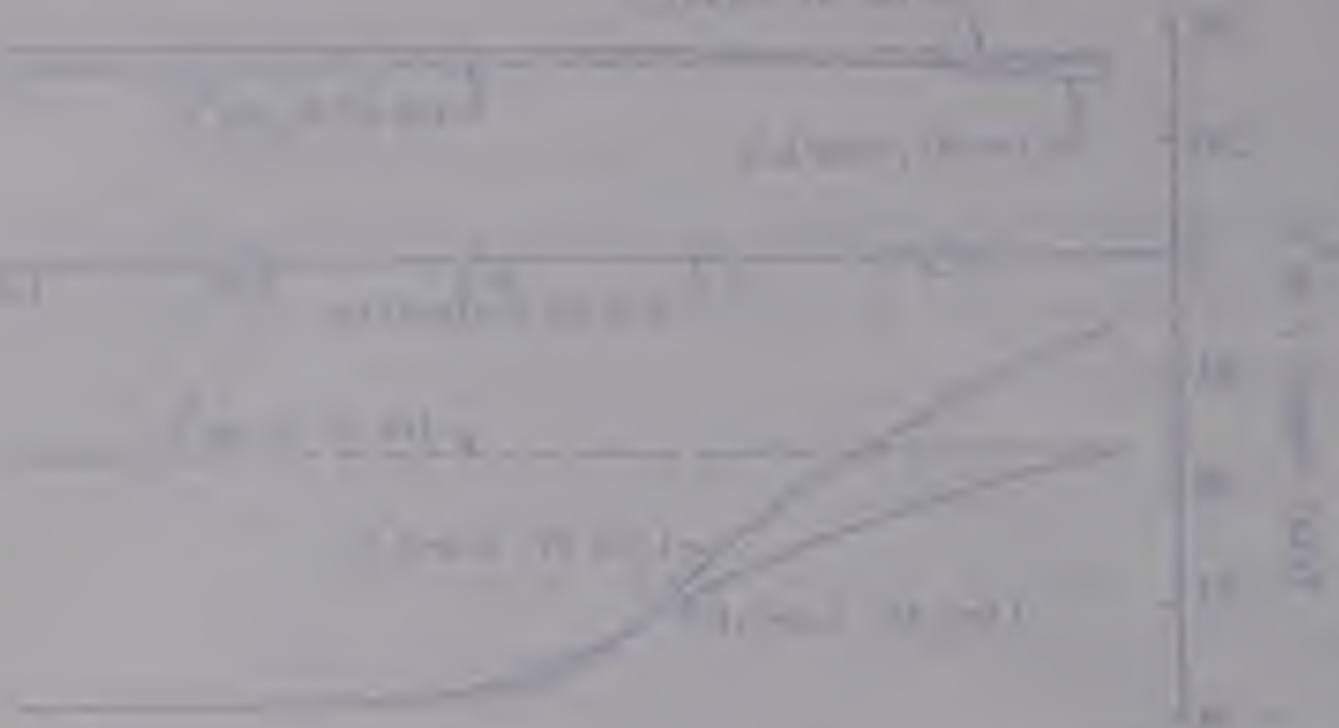
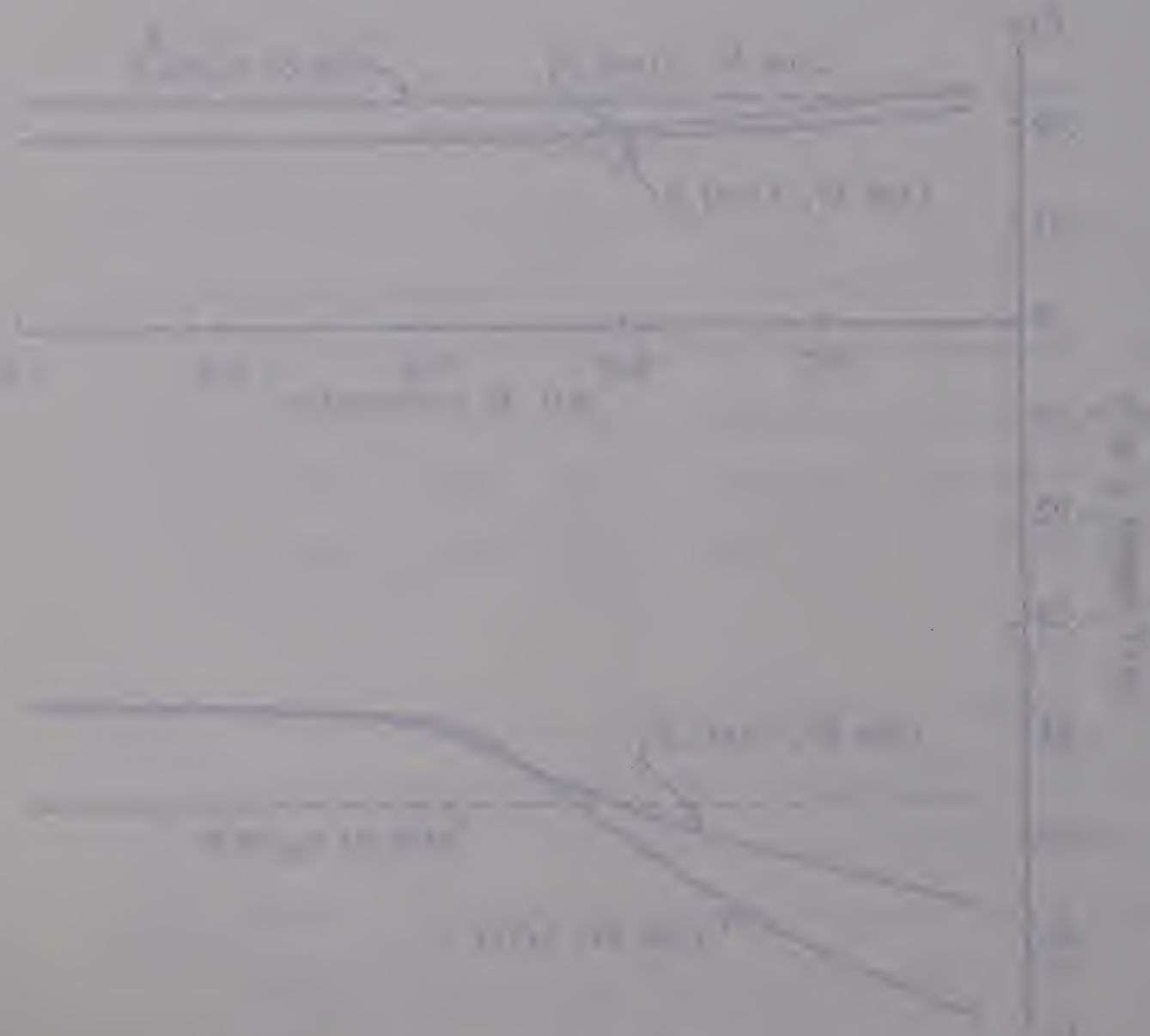


FIGURE 5.6 COLUMN STRIP MOMENTS IN X-DIRECTION
STRIP WIDTH=0.5b





The distribution of negative moment across line 1-Y in FIGURES 4.12 to 4.14 suggest that the distribution of moment into strips should be related to the column aspect ratio. For small values of c/L two strips appear sufficient. However the plots for c/L = 0.4 indicate that the panel should be divided into three strips, but this becomes difficult since the resulting strip definitions would not apply for other column aspect ratios. The plots for c/L = 0.8 indicate that for high values of c/L only two strips again need be defined.

A definite recommendation for the design moments in the x-direction is beyond the scope of this thesis. However preliminary work indicates that the existing design procedures can be used for c/L ratios less than 0.3. For values greater than 0.3 a new definition of slab strips is required.

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

This investigation examined the effect of the column aspect ratio upon the elastic behavior of flat plate slabs. The analysis was restricted to a typical interior panel of an infinite flat plate supported at intervals by non-deflecting columns. Besides the column aspect ratio, other variables examined included the slab aspect ratio, Poisson's ratio, and four different types of loads.

The analysis was carried out using the method of finite differences. The resulting 441 simultaneous equations were solved by a method which took full advantage of the population scarcity in the matrix.

Once the general slab behavior was determined an attempt was made to interpret the results in terms of current design procedures.

6.2 Conclusions

Consistent with the assumptions used, the following are the conclusions drawn from the results of this investigation.

a) Deflections

The maximum slab deflection always occurred at the centre point of the panel. Increasing the column aspect ratio

decreased the maximum deflections. In all cases the deflection reached its minimum value at c/L = 1.0 when the slab was supported along two sides by a wall. Increasing the slab aspect ratio increased the maximum slab deflection significantly for c/L ratios less than 0.5. The general behavior with respect to deflections for line loads and concentrated loads was similar to the behavior of the uniformly distributed load although the magnitudes were different.

b) Bending Moments

A significant beneficial effect in both deflections and bending moments is obtained for even small increases in the column aspect ratio. As the c/L ratio was increased, the maximum positive moments in the y-direction and the negative moments in both directions decreased while the positive moments in the x-direction remained almost constant. The point of inflection along lines in the x-direction was only shifted slightly due to the column aspect ratio. Along lines in the y-direction there was no point of inflection at high c/L values.

An increase in the value of Poisson's ratio from 0.0 to 0.2 was found to effect the slab bending moments significantly. This effect was most pronounced for the maximum positive moment in the x-direction where the moment was increased considerably at low c/L ratios.

The distribution of positive moments across lines in the slab was found to be relatively uniform. The negative moments across the column face in the x-direction were characterized by a high con-

centration of moment in the vicinity of the column corner.

There was no significant changes in behavior which could be attributed to the slab aspect ratio. However, the results for c/L = 0.8, for all values of b/a, were quite similar indicating that for high column aspect ratios the slab behaves primarily in one-way action and is thus independent of the slab aspect ratio.

The magnitudes of the maximum moments under the other three types of loading considered were greater than for the uniform load. The maximum negative moments at the column corner were similar for the cases of a concentrated load at the midpoint of the panel and a line load along the centre line parallel to the x-axis.

c) Shear Distribution Around the Column

The shear distribution around the column was characterized by a high concentration of shear at the column corners. Along lines a/40 and a/20 from the column face the shear was more uniformly distributed although the greatest intensity was still in the vicinity of the column corner. For c/L ratios greater than 0.4 the percent of the total load taken as shear at the column corners decreased linearly until the minimum value was reached at c/L = 1.0 where the shear is uniformly distributed along the wall face. There were no significant changes in shear distribution due to varying the slab aspect ratio.

d) Twisting Moments

The twisting moments around the column were highest in the vicinity of the column corner, decreasing rapidly towards the

centre of the column face.

e) Comparison for Design

The ACI Building Code (318-63) predicts satisfactorily the total design moment in a flat plate slab for all values of the column aspect ratio. In the y-direction as the column aspect ratio increases the Code becomes increasingly conservative. In the x-direction the Code does not consider the transition between two-way and one-way slab action.

With respect to the y-direction the current code procedures for dividing slabs into strips and the portion of M_o assigned to these strips are adequate for all practical purposes. However, the maximum negative moments, in the y-direction, are across the line in the x-direction defined by the column face.

In the x-direction the code procedures are satisfactory for c/L ratios in the y-direction less than approximately 0.3. For values greater than this a new definition of slab strips is required.

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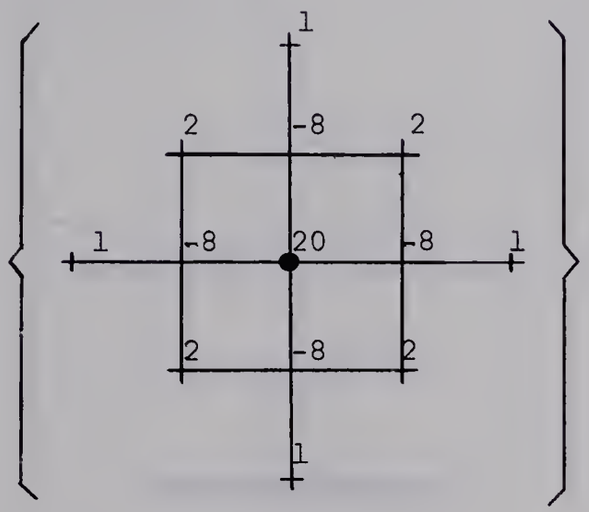
APPENDIX A

FINITE DIFFERENCE EQUATIONS

Listed below are representative examples of the finite difference equations used in this analysis.

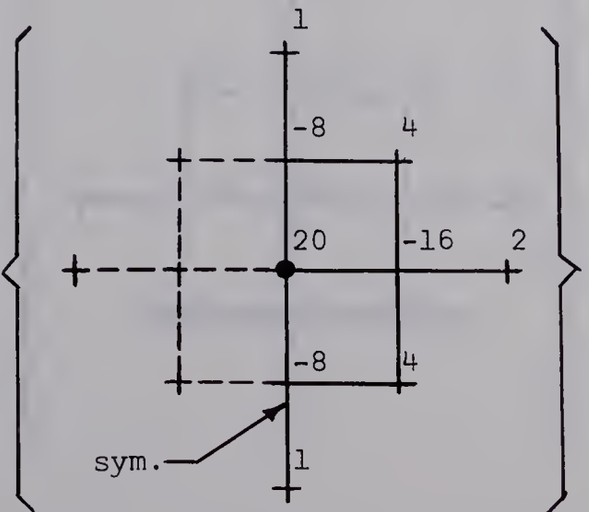
A.1 Deflections

i) General equation



$$w = \frac{qh^4}{D}$$

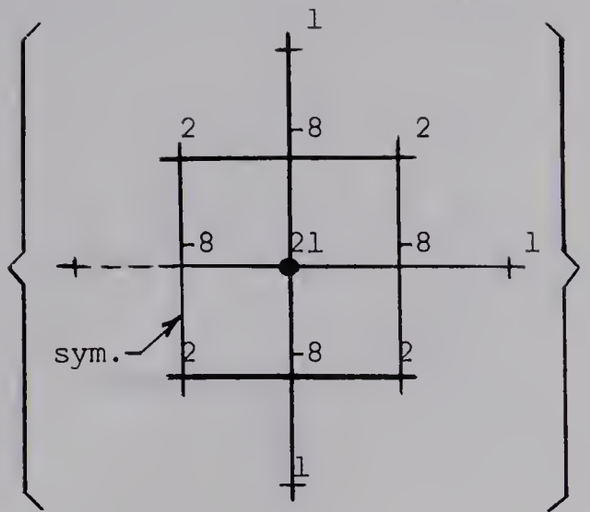
ii) Equation at lines of symmetry



$$w = \frac{qh^4}{D}$$

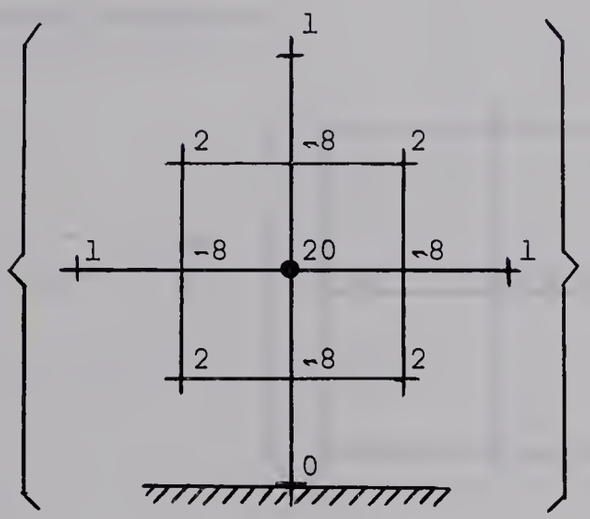




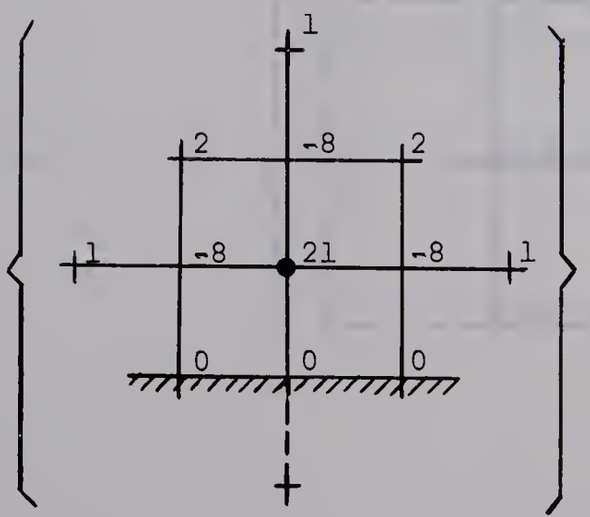


$$w = \frac{qh^4}{D}$$

iii) Equations in the vicinity of the column



$$w = \frac{qh^4}{D}$$

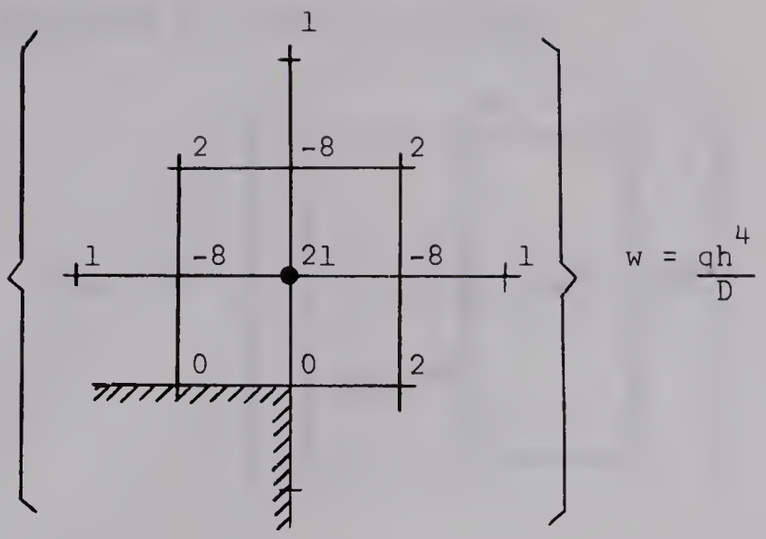


$$w = \frac{qh^4}{D}$$









A.2 Bending Moments

i) General equations

$$m_{ox} = \frac{D}{h} \begin{Bmatrix} & -1 & \\ -\mu & 2+2\mu & -\mu \\ & -1 & \end{Bmatrix} w$$

$$m_{oy} = \frac{D}{h} \begin{Bmatrix} & -\mu & \\ -1 & 2+2\mu & -1 \\ & -\mu & \end{Bmatrix} w$$

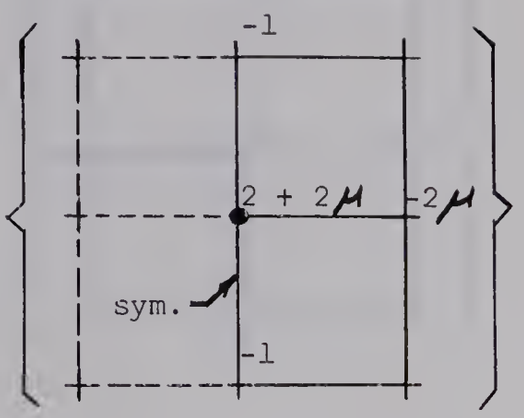






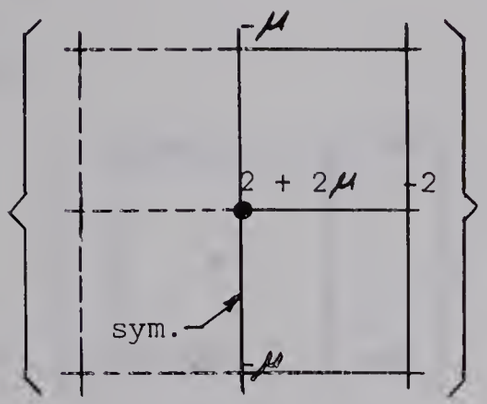
ii) Equations at lines of symmetry

$$m_{ox} = \frac{D}{h}$$



w

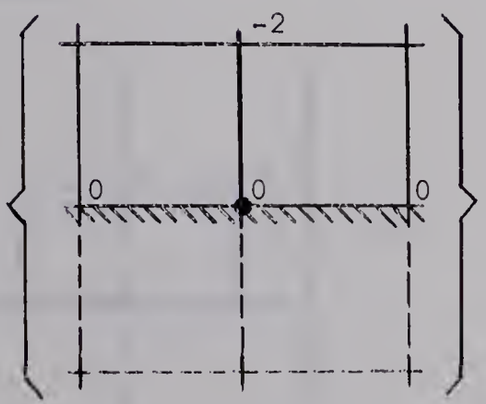
$$m_{oy} = \frac{D}{h}$$



w

iii) Equations in the vicinity of the column

$$m_{ox} = \frac{D}{h}$$



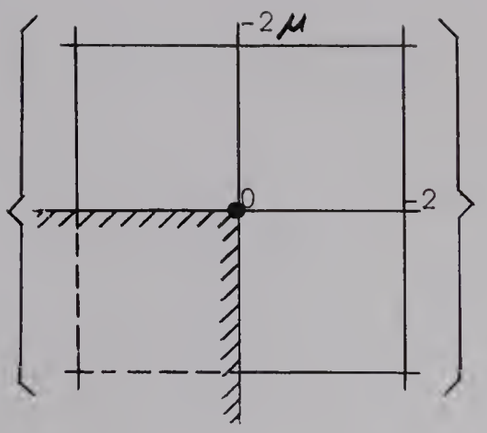
w







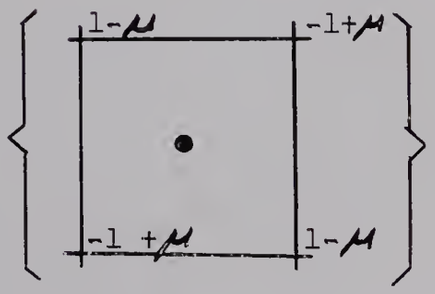
$$m_{oy} = \frac{D}{h}$$



A.3 Twisting Moments

i) General equation

$$m^{o}_{xy} = \frac{D}{h}$$



A.4 Column Shears

i) General expression

$$V_o = \frac{D}{h^2}$$

